Numerical Differentiation

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Abstract

This is a Differentiation to a polynomial so defined for a given data set. And numerically Differentiation is executed.

1 Numerical Differentiation using Newtons forward and Backward Interpolation polynomial

We know Newtons forward polynomial as

$$y = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}$$

$\Delta y_0^3 + \cdots$
(1)

Where $P = \frac{x-x_0}{h}$

$$\frac{dy}{dx} = \frac{dy}{dP}\frac{dP}{dx} = \frac{1}{h}\frac{d}{dP}[y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta y_0^3 + \cdots]$$
(2)

At $x = x_0$

$$\left[\frac{dy}{dx}\right]_{x=x_0} = \frac{1}{h}[\Delta y_0 + \frac{1}{2}$$

$\Delta^2 y_0 + \frac{1}{3}\Delta y_0^3 - \frac{1}{4}\Delta^4 y_0 + \cdots]$(3)

$$y_0' = \frac{1}{h}[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4 + \cdots$$

(4)

Similarly we can do higher Differentiation

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dp}\frac{dy}{dx}\frac{dP}{dx} = \frac{1}{h}\frac{d}{dP}\left[\frac{dy}{dx}\right]$$

$$= \frac{1}{h^2}[\Delta^2 y_0 + \frac{6P-6}{6}^3 y_0 + \frac{12P^2-36P+22}{24}\Delta^4 y_0 + ...$$

$$\left[\frac{d^2y}{dx^2}\right]_{x=x_0} = \frac{1}{h^2}[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + ...]$$

(5)

Similarly one can go on calculating

$$\left[\frac{d^3y}{dx^3}\right]_{x=x_0}, \left[\frac{d^4y}{dx^4}\right]_{x=x_0}, \left[\frac{d^5y}{dx^5}\right]_{x=x_0}, \ldots, \left[\frac{d^ny}{dx^n}\right]_{x=x_0}$$

Home Work: Evalute the above diffrential.
Similarly using New Backward Interpolation formula we have

$$\left[\frac{dy}{dx}\right]_{x=x_n} = \frac{1}{h}\left(\nabla + \frac{1}{2}\nabla^2 + \frac{1}{3}\nabla^3 + \cdots\right) y_n$$

(6)

And

$$\left[\frac{d^2y}{dx^2}\right]_{x=x_n} = \frac{1}{h^2}\left(\nabla^2 + \nabla^3 + \frac{11}{12}\nabla^4 + \cdots\right) y_n$$

(7)

and similarly we can go on calculating

$$\left[\frac{d^3y}{dx^3}\right]_{x=x_n}$$

,

$$\left[\frac{d^4y}{dx^4}\right]_{x=x_n}$$

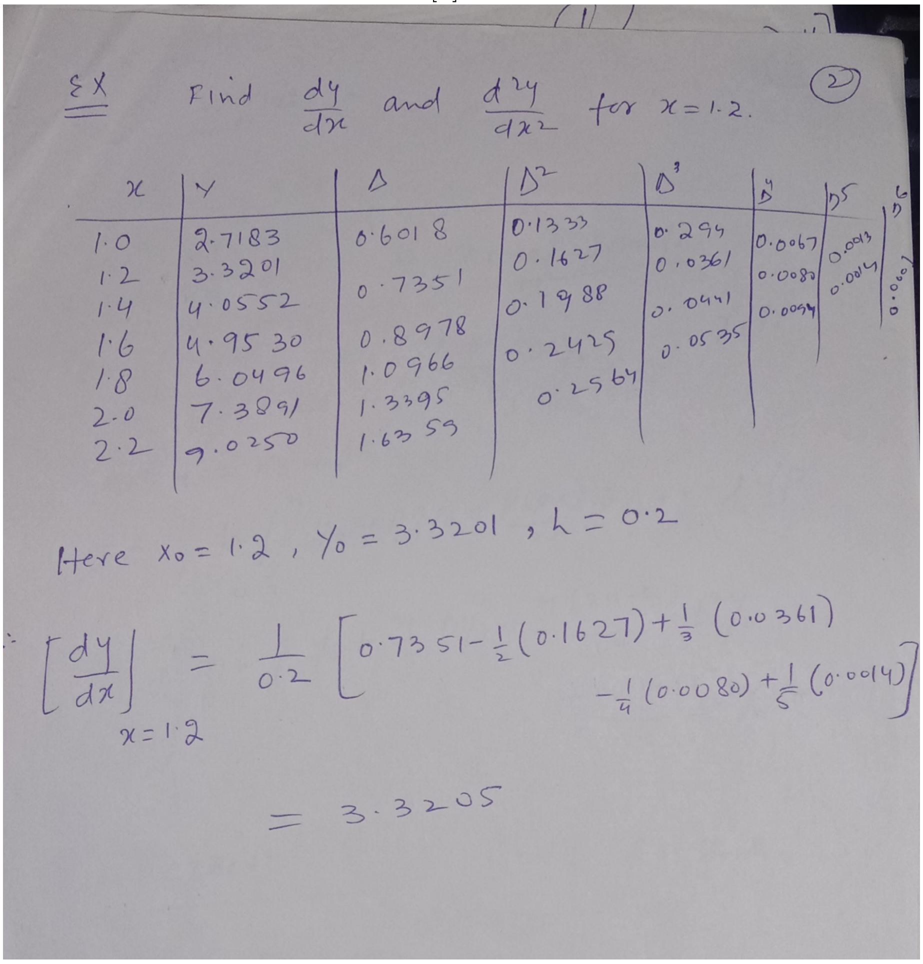
$$\left[\frac{d^5y}{dx^5}\right]_{x=x_n}$$

$$\ldots, \left[\frac{d^n y}{dx^n}\right]_{x=x_n}$$

(8)

Example: Find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ for $x = 1.2$ for the given table.

[h]0.2



Ex Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$. (2)

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
1.0	2.7183	0.6018	0.1333	0.294	0.0067	0.0013	0.000[illegible]
1.2	3.3201	0.7351	0.1627	0.0361	0.0080	0.0014	
1.4	4.0552	0.8978	0.1988	0.0441	0.0094		
1.6	4.9530	1.0966	0.2425	0.0535			
1.8	6.0496	1.3395	0.2564				
2.0	7.3891	1.6359					
2.2	9.0250						

Here $x_0 = 1.2$, $y_0 = 3.3201$, $h = 0.2$

$$\therefore \left[\frac{dy}{dx}\right]_{x=1.2} = \frac{1}{0.2}\left[0.7351 - \frac{1}{2}(0.1627) + \frac{1}{3}(0.0361) - \frac{1}{4}(0.0080) + \frac{1}{5}(0.0014)\right]$$

$$= 3.3205$$

Figure 1: Example