Numerical Integrations

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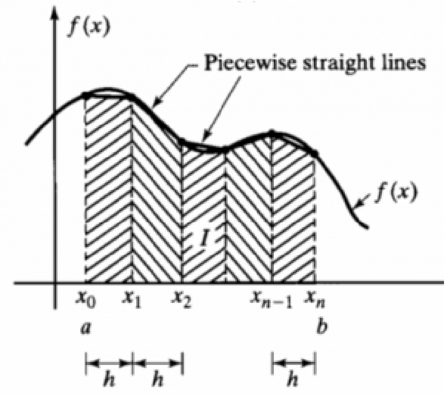
Outline

1. Numerical Integration
   - Area under the curve by Rectangular rule
   - Area under the curve by Mid-Point Rule
   - Area under the curve by Interpolation: Integration by Interpolation

Numerical Integrations



$$I = \int_a^b f(x)dx$$

can be viewed graphically as the area between the x-axis and the curve $y = f(x)$ in the region of the limits of integration a and b

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- and by extension, the term is also sometimes used to describe the numerical solution of differential equations

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- Explore various way for approximating the integrals of a function over a given domain
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- Area under the curve is the value of a integral over a given domain.

Area under the curve by Rectangular rule

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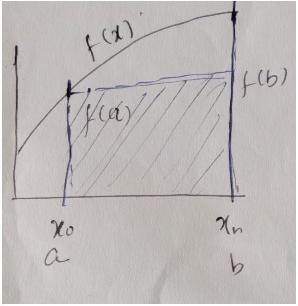
- 

Rectangular rule

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- If $f(x)$ is given as

$$\int_a^b f(x)dx$$

Then Figure shows that the area under the curve is given as rectangle with length $f(a)$ and width $(b - a)$
therefore area is

$$\int_a^b f(x)dx = f(a) \times (b - a)$$

f(a)*(b-a)

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- Both the above mentioned ways contain good amount of error.

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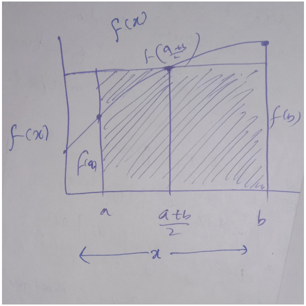
- 

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- evaluate this for total area.

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This time we need to calculate by taking each consecutive da points as

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total area is

$$\text{Area} = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \int_{x_2}^{x_3} f(x)dx + \quad + \int_{x_{n-1}}^{x_n} f(x)dx$$

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$$I = \int_{x_0}^{x_n} [y_0 + p \quad y_0 + \frac{p(p \quad 1)}{2!} \quad {}^2y_0$$

$$+ \frac{p(p \quad 1)(p \quad 2)}{3!} \quad {}^3y_0 + \quad + \frac{p(p \quad 1)(p \quad 2):::((p \quad (n \quad 1)}{n!} \quad {}^ny_0$$

Area by interpolation contd...

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Here

$$\frac{x \quad x_0}{h} = p$$

$$dx = hdp$$

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Frame Title

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Now depending o**n** we have
for n = 1 only two points that is $x_0; x_1$ which gives polynomial as degree 1 , a straight line and $y_0 = y_1 \quad y_0$ and higher di erences will be zero

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In this way we see the beauty of all these formulations. How Interpolation is correctly leading us.

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- the way we calculate from I_1 we can calculate I_2 instead of from x_0 to x_1 this time it is from x_0 to x_2.
- so it will be divided into two parts or trapeziums at x_1 so total area is

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