Numerical Integrations

G N Dar
Department of Physics KU Srinagar

September 9, 2020

Outline

- Error in Trapezium Rule
- Integration by Simpsons's Rule
- Errors in Simpsons one third rule
- Simpsons three Eight Rule ($\frac{3}{8}$ Rule)

Area BY Trapezium Rule

Area BY Trapezium Rule

- $$I = I_1 + I_2 + I_3 + \cdots + I_n = \frac{h}{2}(y_1 + y_0) + \frac{h}{2}(y_2 + y_1)$$
$$+ \frac{h}{2}(y_3 + y_2) + \cdots + \frac{h}{2}(y_n + y_{n-1})$$

Area BY Trapezium Rule

- $$I = I_1 + I_2 + I_3 + \cdots + I_n = \frac{h}{2}(y_1 + y_0) + \frac{h}{2}(y_2 + y_1)$$
  $$+\frac{h}{2}(y_3 + y_2) + \cdots + \frac{h}{2}(y_n + y_{n-1})$$
- Which in turn can be written as

Area BY Trapezium Rule

- $$I = I_1 + I_2 + I_3 + \cdots + I_n = \frac{h}{2}(y_1 + y_0) + \frac{h}{2}(y_2 + y_1)$$
$$+\frac{h}{2}(y_3 + y_2) + \cdots + \frac{h}{2}(y_n + y_{n-1})$$
- Which in turn can be written as
- $$\int_{x_0}^{x_n} y dx = \sum_{i=1}^{n} I_i = \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + y_3 + ... + y_{n-1}]$$

Area BY Trapezium Rule

- $$I = I_1 + I_2 + I_3 + \cdots + I_n = \frac{h}{2}(y_1 + y_0) + \frac{h}{2}(y_2 + y_1)$$
$$+\frac{h}{2}(y_3 + y_2) + \cdots + \frac{h}{2}(y_n + y_{n-1})$$
- Which in turn can be written as
- $$\int_{x_0}^{x_n} ydx = \sum_{i=1}^{n} I_i = \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + y_3 + ... + y_{n-1}]$$
- $$I = \frac{h}{2}[X + 2K]$$

Area BY Trapezium Rule

- $$I = I_1 + I_2 + I_3 + \cdots + I_n = \frac{h}{2}(y_1 + y_0) + \frac{h}{2}(y_2 + y_1)$$
  $$+\frac{h}{2}(y_3 + y_2) + \cdots + \frac{h}{2}(y_n + y_{n-1})$$
- Which in turn can be written as
- $$\int_{x_0}^{x_n} ydx = \sum_{i=1}^{n} I_i = \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + y_3 + ... + y_{n-1}]$$
- $$I = \frac{h}{2}[X + 2K]$$
- where $X = y_0 + y_n$ and $K = \mathrm{y}_1 + y_2 + y_3 + ... + y_{n-1}$

Area BY Trapezium Rule

- $$I = I_1 + I_2 + I_3 + \cdots + I_n = \frac{h}{2}(y_1 + y_0) + \frac{h}{2}(y_2 + y_1)$$
$$+ \frac{h}{2}(y_3 + y_2) + \cdots + \frac{h}{2}(y_n + y_{n-1})$$
- Which in turn can be written as
- $$\int_{x_0}^{x_n} y dx = \sum_{i=1}^{n} I_i = \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + y_3 + ... + y_{n-1}]$$
- $$I = \frac{h}{2}[X + 2K]$$
- where $X = y_0 + y_n$ and $K = \mathrm{y}_1 + y_2 + y_3 + ... + y_{n-1}$
- We call it all a *Trapezium Rule*

Error in Trapezium Rule

Error in Trapezium Rule

- We calculate the area under each trapezium using Taylor expansion

Error in Trapezium Rule

- We calculate the area under each trapezium using Taylor expansion
- which is expected to be more real or having less error and then

Error in Trapezium Rule

- We calculate the area under each trapezium using Taylor expansion
- which is expected to be more real or having less error and then
- compare the above evaluated $I_1, I_2, ...I_n$ Results.

Error in Trapezium Rule

- We calculate the area under each trapezium using Taylor expansion
- which is expected to be more real or having less error and then
- compare the above evaluated $I_1, I_2, ...I_n$ Results.
- Let $f(x)$ be continuous well behaved posses derivative in $[x_0, x_n]$. Expand y in a Taylor series around $x = x_0$, we get

Error in Trapezium Rule

- We calculate the area under each trapezium using Taylor expansion
- which is expected to be more real or having less error and then
- compare the above evaluated $I_1, I_2, ...I_n$ Results.
- Let $f(x)$ be continuous well behaved posses derivative in $[x_0, x_n]$. Expand y in a Taylor series around $x = x_0$, we get
- $$\int_{x_0}^{x_1} y dx = \int_{x_0}^{x_1} [y_0 + (x - x_0)y_0' + \frac{(x - x_0)^2}{2!} y_0'' + \cdots] dx$$

Error in Trapezium rule

Error in Trapezium rule

- $$= y_0 h + \frac{h^2}{2} y_0' + \frac{h^3}{6} y_0'' + \frac{h^4}{24} y_0'' + \cdots$$

Error in Trapezium rule

- $$= y_0 h + \frac{h^2}{2} y_0' + \frac{h^3}{6} y_0'' + \frac{h^4}{24} y_0'' + \cdots$$
- Similarly expanding the previous result obtained using Interpolation for

Error in Trapezium rule

- $$= y_0 h + \frac{h^2}{2} y_0' + \frac{h^3}{6} y_0'' + \frac{h^4}{24} y_0'' + \cdots$$
- Similarly expanding the previous result obtained using Interpolation for
- $$I_1 = \frac{h}{2}(y_0 + y_1)$$

Error in Trapezium rule

- $$= y_0 h + \frac{h^2}{2} y_0' + \frac{h^3}{6} y_0'' + \frac{h^4}{24} y_0'' + \cdots$$
- Similarly expanding the previous result obtained using Interpolation for
- $$I_1 = \frac{h}{2}(y_0 + y_1)$$
- we get after expanding $y_1 = f(x_0 + h)$ around x_0

Error in Trapezium rule

Error in Trapezium rule

- $$= \frac{h}{2}[y_0 + y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \cdots]$$

Error in Trapezium rule

- $$= \frac{h}{2}[y_0 + y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \cdots]$$
- Both the above equation represent the same thing so the error is going to be just difference AFTER NEGLECTING HIGHER ORDER TERMS

Error in Trapezium rule

- $$= \frac{h}{2}[y_0 + y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \cdots]$$
- Both the above equation represent the same thing so the error is going to be just difference AFTER NEGLECTING HIGHER ORDER TERMS
- $$Error = \frac{h^3}{6}y_0'' - \frac{h^3}{4}y_0'' + ... = -\frac{1}{12}h^3[y_0]$$

Error in Trapezium rule

Error in Trapezium rule

- Error $\propto h^3$

Error in Trapezium rule

- Error $\propto h^3$
- Total error will be summation of n number of errors(trapeziums) FROM $x_0 - x_n$

Error in Trapezium rule

- Error $\propto h^3$
- Total error will be summation of n number of errors(trapeziums) FROM $x_0 - x_n$
- Error Total

$$E = E_1 + E_2 + E_3 + \ldots\ldots\ldots\ldots + E_n$$

Error in Trapezium rule

- Error $\propto h^3$
- Total error will be summation of n number of errors(trapeziums) FROM $x_0 - x_n$
- Error Total

$$E = E_1 + E_2 + E_3 + ... + E_n$$

- $$= -\frac{1}{12}h^3[y_0'' + y_1'' + y_2'' + ... + y_{n-1}$$

Error in Trapezium rule

Error in Trapezium rule

- Assume $y(\bar{x})''$ is the largest value of n quantities in the bracket then we can write

Error in Trapezium rule

- Assume $y(\bar{x})''$ is the largest value of n quantities in the bracket then we can write
- $$= -\frac{1}{12}h^3 n[y(\bar{x})'']$$

  As we know

  $$(x_n - x_0) = nh = (b - a) = nh$$

  finally

Error in Trapezium rule

- Assume $y(\bar{x})''$ is the largest value of n quantities in the bracket then we can write
- $$= -\frac{1}{12}h^3 n[y(\bar{x})'']$$

  As we know
  $$(x_n - x_0) = nh = (b - a) = nh$$

  finally
- $$= -\frac{1}{12}(x_n - x_0)h^2 y''(\bar{x})$$

Error in Trapezium rule

- Assume $y(\bar{x})''$ is the largest value of n quantities in the bracket then we can write
- $$= -\frac{1}{12}h^3 n[y(\bar{x})'']$$

  As we know

  $$(x_n - x_0) = nh = (b - a) = nh$$

  finally
- $$= -\frac{1}{12}(x_n - x_0)h^2 y''(\bar{x})$$
- Error $\propto h^2$

Integration by Simpsons's Rule

Integration by Simpsons's Rule

- The rule is obtained by putting $n = 2$ into the main equation under integration by interpolation

Integration by Simpsons's Rule

- The rule is obtained by putting $n = 2$ into the main equation under integration by interpolation
- so instead of straight lines in the trapezium we now take care of curved nature so that errors will be reduced further.

Integration by Simpsons's Rule

- The rule is obtained by putting $n = 2$ into the main equation under integration by interpolation
- so instead of straight lines in the trapezium we now take care of curved nature so that errors will be reduced further.
- Starting again from the main equation

Integration by Simpsons's Rule

- The rule is obtained by putting $n = 2$ into the main equation under integration by interpolation
- so instead of straight lines in the trapezium we now take care of curved nature so that errors will be reduced further.
- Starting again from the main equation
- $$I = \int_0^n h[y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$$
$$+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots + \frac{p(p-1)(p-2)...((p-(n-1)}{n!}\Delta^n y_0]dp$$

Integration by Simpsons's Rule

- The rule is obtained by putting $n = 2$ into the main equation under integration by interpolation
- so instead of straight lines in the trapezium we now take care of curved nature so that errors will be reduced further.
- Starting again from the main equation
- $$I = \int_0^n h[y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$$
$$+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots + \frac{p(p-1)(p-2)...((p-(n-1)}{n!}\Delta^n y_0]dp$$
- $$I = \int_{x_0}^{x_2} ydx = nh[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24} + \Delta^3 y_0 + \cdots]$$

Integration by Simpsons's Rule

Integration by Simpsons's Rule

- for $n = 2$, together we take x_0, x_1, x_2 this second difference will exist and all higher differences will vanish

Integration by Simpsons's Rule

- for $n = 2$, together we take x_0, x_1, x_2 this second difference will exist and all higher differences will vanish
- $$I = \int_{x_0}^{x_2} ydx = 2h[y_0 + \frac{2}{2}\Delta y_0 + \frac{2(2*2-3)}{12}\Delta^2 y_0$$
  $$+\frac{2(2-2)^2}{24} + \Delta^3 y_0 + \cdots]$$

Integration by Simpsons's Rule

- for $n = 2$, together we take x_0, x_1, x_2 this second difference will exist and all higher differences will vanish
-
$$I = \int_{x_0}^{x_2} ydx = 2h[y_0 + \frac{2}{2}\Delta y_0 + \frac{2(2*2-3)}{12}\Delta^2 y_0$$
$$+\frac{2(2-2)^2}{24} + \Delta^3 y_0 + \cdots]$$
-
$$I = 2h[y_0 + \frac{2}{2}(y_1 - y_0) + \frac{1}{6}(y_2 - 2y_1 + y_0)$$

Integration by Simpsons's Rule

- for $n = 2$, together we take x_0, x_1, x_2 this second difference will exist and all higher differences will vanish
- $$I = \int_{x_0}^{x_2} y dx = 2h[y_0 + \frac{2}{2}\Delta y_0 + \frac{2(2*2-3)}{12}\Delta^2 y_0 + \frac{2(2-2)^2}{24} + \Delta^3 y_0 + \cdots]$$
- $$I = 2h[y_0 + \frac{2}{2}(y_1 - y_0) + \frac{1}{6}(y_2 - 2y_1 + y_0)$$
- $$I = I_1 = \frac{h}{3}[y_0 + 4y_1 + y_2]$$

Integration by Simpsons's Rule

Integration by Simpsons's Rule

- Similarly for $x_2 \to x_4, x_4 \to x_6, x_6 \to x_8, \cdots, x_{n-2} \to x_n$ strips we get

Integration by Simpsons's Rule

- Similarly for $x_2 \rightarrow x_4, x_4 \rightarrow x_6, x_6 \rightarrow x_8, \cdots, x_{n-2} \rightarrow x_n$ strips we get
- $$I = I_2 = \frac{h}{3}[y_2 + 4y_3 + y_4]$$

Integration by Simpsons's Rule

- Similarly for $x_2 \to x_4, x_4 \to x_6, x_6 \to x_8, \cdots, x_{n-2} \to x_n$ strips we get
- $$I = I_2 = \frac{h}{3}[y_2 + 4y_3 + y_4]$$
- $$I = I_3 = \frac{h}{3}[y_4 + 4y_5 + y_6]$$

Integration by Simpsons's Rule

- Similarly for $x_2 \to x_4, x_4 \to x_6, x_6 \to x_8, \cdots, x_{n-2} \to x_n$ strips we get
- $$I = I_2 = \frac{h}{3}[y_2 + 4y_3 + y_4]$$
- $$I = I_3 = \frac{h}{3}[y_4 + 4y_5 + y_6]$$
- $$I = I_4 = \frac{h}{3}[y_6 + 4y_7 + y_8]$$

$$\vdots$$

$$I = I_{n-1} = \frac{h}{3}[y_{n-2} + 4y_{n-1} + y_n]$$

Integration by Simpsons's Rule

Integration by Simpsons's Rule

- Total area under the curve will be just summations of all the above integrals

Integration by Simpsons's Rule

- Total area under the curve will be just summations of all the above integrals
- $$I_T = \int_{x_0}^{x_n} y dx = \sum_{i=1}^{n-1} I_i$$

$$= \frac{h}{3}[y_0 + 4(y_1 + y_3 + y_5 + \cdots + y_{2n-1}) + 2(y_2 + y_4 + y_6 + \cdots + y_{2n-2}]$$

Integration by Simpsons's Rule

- Total area under the curve will be just summations of all the above integrals
- $$I_T = \int_{x_0}^{x_n} y dx = \sum_{i=1}^{n-1} I_i$$
  $$= \frac{h}{3}[y_0 + 4(y_1 + y_3 + y_5 + \cdots + y_{2n-1}) + 2(y_2 + y_4 + y_6 + \cdots + y_{2n-2}]$$
- $$= \frac{h}{3}[X + 4O + 2E]$$
  where the symbols have self explanatory meaning.

Errors in Simpsons one third rule

Errors in Simpsons one third rule

- Expanding $y(x)$ using Taylor and finding the result

Errors in Simpsons one third rule

- Expanding $y(x)$ using Taylor and finding the result
- $$I = \int_{x_0}^{x_2} y dx = \int_{x_0}^{(x_0+2h)} y dx$$

Errors in Simpsons one third rule

- Expanding $y(x)$ using Taylor and finding the result
- $$I = \int_{x_0}^{x_2} ydx = \int_{x_0}^{(x_0+2h)} ydx$$
- $$= \int_{x_0}^{(x_0+2h)} [y_0 + (x - x_0)y_0' + \frac{(x - x_0)^2}{2!} y_0'' + \cdots]dx$$

Errors in Simpsons one third rule

- Expanding $y(x)$ using Taylor and finding the result
- $$I = \int_{x_0}^{x_2} y dx = \int_{x_0}^{(x_0+2h)} y dx$$
- $$= \int_{x_0}^{(x_0+2h)} [y_0 + (x - x_0)y_0' + \frac{(x - x_0)^2}{2!} y_0'' + \cdots]dx$$
- $$= 2hy_0 + 2h^2 y_0' + \frac{4}{3} h^3 y_0'' + \frac{2}{3} h^4 y_0''' + \cdots$$

  while as from the different method we got as below for x_0 to x_2

Errors in Simpsons one third rule

- Expanding $y(x)$ using Taylor and finding the result
- $$I = \int_{x_0}^{x_2} ydx = \int_{x_0}^{(x_0+2h)} ydx$$
- $$= \int_{x_0}^{(x_0+2h)} [y_0 + (x - x_0)y_0' + \frac{(x - x_0)^2}{2!}y_0'' + \cdots]dx$$
- $$= 2hy_0 + 2h^2y_0' + \frac{4}{3}h^3y_0'' + \frac{2}{3}h^4y_0''' + \cdots$$

  while as from the different method we got as below for x_0 to x_2
- $$I = I_1 = \frac{h}{3}[y_0(x) + 4y_1(x + h) + y_2(x + 2h)]$$

Errors contd using Simpsons rule....

Errors contd using Simpsons rule....

- Now Expand it again around x_0 as Taylor series

Errors contd using Simpsons rule....

- Now Expand it again around x_0 as Taylor series
- $$= \frac{h}{3}[(y_0 + 4(y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \cdots) + (y_0 + 2hy_0' + \frac{4h^2}{2!}y_0'' + \cdots)]$$

Errors contd using Simpsons rule....

- Now Expand it again around x_0 as Taylor series
- $$= \frac{h}{3}[(y_0 + 4(y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \cdots) + (y_0 + 2hy_0' + \frac{4h^2}{2!}y_0'' + \cdots)]$$
- $$= 2hy_0 + 2h^2y_0' + \frac{4}{3}h^3y_0'' + \frac{2h^4}{3}y_0''' + \frac{5}{18}h^5y_0''''$$

Errors contd using Simpsons rule....

- Now Expand it again around x_0 as Taylor series
- $$= \frac{h}{3}[(y_0 + 4(y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \cdots) + (y_0 + 2hy_0' + \frac{4h^2}{2!}y_0'' + \cdots)]$$
- $$= 2hy_0 + 2h^2y_0' + \frac{4}{3}h^3y_0'' + \frac{2h^4}{3}y_0''' + \frac{5}{18}h^5y_0''''$$
- Therefore error E_1 in first strip is

Errors contd using Simpsons rule....

- Now Expand it again around x_0 as Taylor series
- $$= \frac{h}{3}[(y_0 + 4(y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \cdots)$$
$$+(y_0 + 2hy_0' + \frac{4h^2}{2!}y_0'' + \cdots)]$$
- $$= 2hy_0 + 2h^2y_0' + \frac{4}{3}h^3y_0'' + \frac{2h^4}{3}y_0''' + \frac{5}{18}h^5y_0''''$$
- Therefore error E_1 in first strip is
- $$E_1 = \int_{x_0}^{x_2} ydx - \frac{h}{3}(y_0 + 4y_1 + y_2) = (\frac{4}{15} - \frac{5}{18})h^5y_0'''' + \ldots = -\frac{h^5}{90}y_0''''$$

Errors contd using Simpsons rule....

Errors contd using Simpsons rule....

- similarly

$$E_2 = -\frac{h^5}{90} y_2''''$$

Errors contd using Simpsons rule....

- similarly

$$E_2 = -\frac{h^5}{90} y_2''''$$

-

$$E_3 = -\frac{h^5}{90} y_4''''$$

Errors contd using Simpsons rule....

- similarly

$$E_2 = -\frac{h^5}{90} y_2^{''''}$$

-

$$E_3 = -\frac{h^5}{90} y_4^{''''}$$

-

$$E_4 = -\frac{h^5}{90} y_6^{''''}$$

Errors contd using Simpsons rule....

- similarly

$$E_2 = -\frac{h^5}{90} y_2''''$$

- $$E_3 = -\frac{h^5}{90} y_4''''$$

- $$E_4 = -\frac{h^5}{90} y_6''''$$

- Finally total error will be as

Errors contd using Simpsons rule....

- similarly

$$E_2 = -\frac{h^5}{90} y_2''''$$

-

$$E_3 = -\frac{h^5}{90} y_4''''$$

-

$$E_4 = -\frac{h^5}{90} y_6''''$$

- Finally total error will be as
-

$$E_T = -\frac{(x_n - x_0)}{180} h^3 y_(\bar{x})''''$$

Errors contd using Simpsons rule....

- similarly

$$E_2 = -\frac{h^5}{90} y_2^{''''}$$

-

$$E_3 = -\frac{h^5}{90} y_4^{''''}$$

-

$$E_4 = -\frac{h^5}{90} y_6^{''''}$$

- Finally total error will be as
-

$$E_T = -\frac{(x_n - x_0)}{180} h^3 y_{(}\bar{x})^{''''}$$

Simpsons three Eight Rule ($\frac{3}{8}$ Rule)

Simpsons three Eight Rule ($\frac{3}{8}$ Rule)

- on Setting $n = 3$ we get first strip from x_0 to x_3

Simpsons three Eight Rule ($\frac{3}{8}$ Rule)

- on Setting $n = 3$ we get first strip from x_0 to x_3
- $$I_1 = \int_{x_0}^{x_3} y dx = \frac{3}{8} h(y_0 + 3y_1 + 3y_2 + y_3)$$

Simpsons three Eight Rule ($\frac{3}{8}$ Rule)

- on Setting $n = 3$ we get first strip from x_0 to x_3
- $$I_1 = \int_{x_0}^{x_3} y dx = \frac{3}{8} h(y_0 + 3y_1 + 3y_2 + y_3)$$
- $$I_2 = \int_{x_3}^{x_6} y dx = \frac{3}{8} h(y_3 + 3y_4 + 3y_5 + y_6)$$

Simpsons three Eight Rule ($\frac{3}{8}$ Rule)

- on Setting $n = 3$ we get first strip from x_0 to x_3
- $$I_1 = \int_{x_0}^{x_3} y dx = \frac{3}{8} h(y_0 + 3y_1 + 3y_2 + y_3)$$
- $$I_2 = \int_{x_3}^{x_6} y dx = \frac{3}{8} h(y_3 + 3y_4 + 3y_5 + y_6)$$
- $$I_3 = \int_{x_6}^{x_9} y dx = \frac{3}{8} h(y_6 + 3y_7 + 3y_8 + y_9)$$

. . . . .

$$I_n = \int_{x_{n-3}}^{x_n} y dx = \frac{3}{8} h(y_{3n-3} + 3y_{3n-2} + 3y_{3n-1} + y_{3n})$$

Simpsons three Eight Rule ($\frac{3}{8}$ Rule)

Simpsons three Eight Rule ($\frac{3}{8}$ Rule)

- Adding up all strips the total area under the function $y(x)$ or integration from x_0 to x_n is

Simpsons three Eight Rule ($\frac{3}{8}$ Rule)

- Adding up all strips the total area under the function $y(x)$ or integration from x_0 to x_n is
- $$I_T = \frac{3}{8}h[y_0 + 3(y_1 + y_2 + y_{y4} + y_5 + \cdots + y_{3n-2} + y_{3n-1})$$
  $$+2(y_3 + y_6 + y_9 + \cdots + y_{3n-3} + y_{3n})]$$

Simpsons three Eight Rule ($\frac{3}{8}$ Rule)

- Adding up all strips the total area under the function $y(x)$ or integration from x_0 to x_n is
- $$I_T = \frac{3}{8}h[y_0 + 3(y_1 + y_2 + y_{y4} + y_5 + \cdots + y_{3n-2} + y_{3n-1})$$ $$+2(y_3 + y_6 + y_9 + \cdots + y_{3n-3} + y_{3n})]$$
- And in this similar to previous cases the dominant error is $$E = -\frac{3}{80}h^5 y(\bar{x})''''$$

Assignment: Determine the erorr for above case

Assignment: Determine the erorr for above case

- *Examples*
  Find the area from $x = 7.47$ to $x = 7.52$ for the given table using rectangle rule , mid point rule, Trapezium rule, Simpsons both the rules.

x	7.47	7.48	7.49	7.50	7.51	7.52
y	1.93	1.95	1.98	2.01	2.03	2.06

Hint: Area by Trapizium rule is 0.0996

Assignment

Examples Solve with $h = 0.5, 0.25, 0.125$

$$I = \int_0^1 \left(\frac{1}{1+x}\right)$$

Assignment

Examples Solve with $h = 0.5, 0.25, 0.125$

$$I = \int_0^1 (\frac{1}{1+x})$$

For $h = 0.5$ we solve it as

x	y
0.0	1.0000
0.5	0.6667
1.0	0.50000

Assignment

Examples Solve with $h = 0.5, 0.25, 0.125$

$$I = \int_0^1 (\frac{1}{1+x})$$

For $h = 0.5$ we solve it as

x	y
0.0	1.0000
0.5	0.6667
1.0	0.50000

Therefore using trapezium rule

$$I = \frac{h}{2}[y_0 + 2y_1 + y_2] = 0.70835$$

using $h = 0.25, I = 0.6970$ and for $h = 0.125, I = 0.6941$

Assignment

Assignment

- Solve the same using Simpsons rules, we will get

Assignment

- Solve the same using Simpsons rules, we will get
- Hint using $h = 0.5, I = 0.6945$ and for $h = 0.25, I = 0.69321$ $h = 0.125, I = 0.6932$

Assignment

- Solve the same using Simpsons rules, we will get
- Hint using $h = 0.5, I = 0.6945$ and for $h = 0.25, I = 0.69321$ $h = 0.125, I = 0.6932$
- Also solve analytically

$$\int_0^1 \frac{1}{1+x} dx = \log_e 2 = 0.693147$$