Interpolations

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August 29, 2020

Outline

1. Interpolations
   - How to do Interpolations: Motivations
   - **Methods of finding a function (Polynomial)**
   - **Direct Method**
   - Interpolation Techniques: Langrange Interpolation
     - Langrane: Quadratic Interpolation
     - Interpolation:
   - Assignment
   - Issues with Langrange interpolation formula
   - Forward Difference Table
   - Backward Difference Table
2. Newtons forward interpolation formula (NFIF)
3. Newtons Backward interpolation formula

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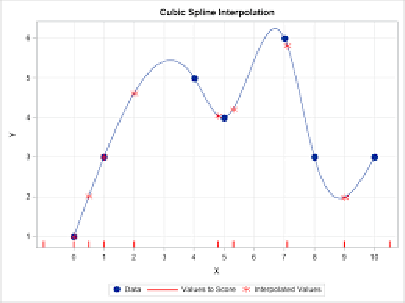
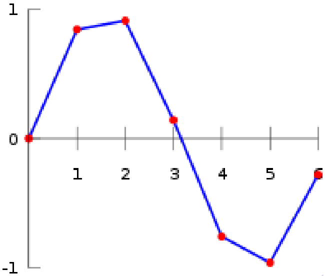
- Each method involves some amount of error and looking or exploring for improvement gives birth to a new set of techniques or methods.

Interpolation





cubic interpolation

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- $$f(x_0) = y_0, f(x_1) = y_1, f(x_2) = y_2, \cdots f(x_n) = y_n$$

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- The process involves to find the coefficients of fitted polynomial using given data set.

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- and n^{th} degree polynomial using $n + 1$ data points

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- $P_n(x_i) = y_i$ for each i , we can write for example

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- $$\begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & x_2^n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

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- Let x_i and y_i is given as $x_i = 1, 2, 3$ and $y_i = 1, 4, 9$. Fit a polynomial through direct method. Write your expression and find $p_n(x)$ at $x = 4$

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- *Langrange* idea is to multiply each f_j by a *polynomial* that is just 1 at x_j and just 0 elsewhere other than x_j .
- Then take sum of these $n + 1$ *polynomials* to get the *unique* interpolation formula of degree n or else. Let us work it out in steps.

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- $L_2(x) = 1$ or o at $x = x_2$ and elesewhere x_2
- The only choice to have the above is as

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- Note that $L_k(x_j) = 0$ if $k \neq j$ otherwise 1

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- $$l_1(x) = (x - x_0)(x - x_2)(x - x_3) \cdots (x - x_n)$$
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- Also note that or check

$$P_n(x_0) = y_0$$

$$P_n(x_1) = y_1$$

$$P_n(x_n) = y_n$$

Assignment

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- For the given table find the Langrange interpolation formula

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- Show that

$$P_3(x) = \frac{5}{3}x^3 - \frac{4}{3}x^2 + 2$$

Verify $P_3(x_0) = y_0 = -1$

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- Accordingly we have three types of Newtons interpolation methods defined through difference tables, taken as

Difference Tables

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- Forward differences

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- difference of dependent variable is involved say $y_i - y_{i+1}$ as per the tables .

Forward Difference Table

Forward Difference Table

-

x	y	Δ (first difference)	Δ^2 (second difference)	Δ^3(third difference)
x_0	y_0			
x_1	y_1	$\Delta y_0 = y_1 - y_0$		
x_2	y_2	$\Delta y_1 = y_2 - y_1$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	
x_3	y_3	$\Delta y_2 = y_3 - y_2$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$

Backward Difference Table

Backward Difference Table

- **Note: Symbolic representation y_0 which is subtracted.**

x	y	∇ (first difference)	∇^2 (second difference)	∇^3(third difference)
$\mathbf{x}_0$	$\mathbf{y}_0$			
$\mathbf{x}_1$	$\mathbf{y}_1$	$\nabla y_1 = y_1 - y_0$		
$\mathbf{x}_2$	$\mathbf{y}_2$	$\nabla y_2 = y_2 - y_1$	$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$	
$\mathbf{x}_3$	$\mathbf{y}_3$	$\nabla y_3 = y_3 - y_2$	$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$	$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$

Assignment

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- **Note: Symbolic representation** y_1, y_2, y_3 **from which subtracted**
  Assignment: choose your data from x_0, y_0 **to** x_6, y_6**. Derive the result for**
  $\Delta y_0, \Delta y_4, \Delta^2 y_0, \Delta y_4, \Delta y_5, \Delta^4 y_0,$

And for

$\nabla y_1, \nabla y_4, \nabla^2 y_2, \nabla^3 y_3$

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Note:

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- **Backward difference table ends at y_n , the last data point used for Backward difference interpolation formula.**

Newtons forward interpolation formula (NFIF)

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$$P_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots \frac{p(p-1)(p-2)....(p-(n-1))}{n!}\Delta^n y_0$$

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- where $p = \frac{x-x_0}{h}$, h a uniform spacing between arguments.

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x=x_0, $P_n(x_0) = a_0 = y_0$

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-

$$\text{x=x}_1,\ P_n(x_1) = y_1 = a_0 + a_1(x_1 - x_0) = y_0 + a_1 * h$$

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- therefore with all these coefficients so evaluated we end up with an expression involving various difference operators and y_0 called as Newtons forward interpolation polynomial as

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- $$P_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots + \frac{p(p-1)\cdots(p-(n-1))}{n!}\Delta^n y_0$$

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- where $p = \frac{x-x_0}{h}$
  The above formula is also known as Newton -Gregory forward polynomial. This is like a Binomial.

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- Write NFIF and substitute the values from table and find the polynomial.

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- $$P_n(x) = y_n + (x - x_n)\frac{\nabla y_n}{h} + \frac{1}{2!}(x - x_n)(x - x_{n-1})\frac{\nabla^2 y_n}{h^2} + \frac{1}{n!}(x - x_n)(x - x_{n-1})...(x - x_1)\frac{\nabla^n y_n}{h^n}$$

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- Note: As the point of estimate lies near end so NBIF will be the most suitable. Please evaluate the result by using NFIF also. Answer: $P_n(7.5) = 421.875$, Since from data it is clear that the table is showing cubic behavior which cab be checked also by calculating $(7.5)^3$

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    - Answer: for 1955 = 54.85 And for 1985 = 96.84