Interpolations

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August 29, 2020

Outline

Central Difference

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- similar higher order central difference can be defined

Central Differences Table

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x	y	δ (first difference	δ^2 (sec.diff)	δ^3 (third diff)	δ^4 (fou
x_0	y_0				
x_1	y_1	$\delta y_{\frac{1}{2}}$			
x_2	y_2	$\delta y_{\frac{3}{2}}$	$\delta^2 y_1$		
x_3	y_3	$\delta y_{\frac{5}{2}}$	$\delta^2 y_2$	$\delta^3 y_{\frac{3}{2}}$	
x_4	y_4	$\delta y_{\frac{7}{2}}$	$\delta^2 y_3$	$\delta^3 y_{\frac{5}{2}}$	δ^4

Table: Central Difference Table

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$$\delta^n y_{r-\frac{1}{2}} = \delta^{n-1} y_r - \delta^{n-1} y_{r-1}$$

for odd n and $r = 1, 2, 3....$ so we have with this $\delta, \delta^2, \delta^3, \delta^4, ...$

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$$\delta^n y_r = \delta^{n-1} y_{r+\frac{1}{2}} - \delta^{n-1} y_{r-\frac{1}{2}}$$

- for even n and $r = 1, 2, 3....$ so we have with this $\delta^2, \delta^4, \delta^6, \delta^8, ...$

Gauss Central Differences interpolation

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- Home work: Derive Gauss central difference interpolation formula.

Symbolic Relations between the operators

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- In addition to forward difference operator Δ, Backward difference operator ∇ and central difference operator δ, we have shift operator $E^{\pm}$ which raises or lowers the change in unit 1 or h depending on the sign.

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  Similar with Backward difference operator ∇ we have

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- Verify them all.

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- $$\delta y_x = y_{x+\frac{h}{2}} - y_{x-\frac{h}{2}} = [E^{\frac{1}{2}} - E^{\frac{-1}{2}}]y_x$$

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- $$\delta = E^{\frac{1}{2}} - E^{\frac{-1}{2}}$$
- we can also prove
$$\Delta = \delta E^{\frac{1}{2}}$$
- we can also show
$$Ef(x) = f(x+h) = e^{hD}f(x)$$
and using Taylor expansion we can define as
$$E = e^{hD}$$

Assignment

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- Note Prove all these relations.
  Assignment. See how difference table is used for detection of errors.
  2. Study Gauss interpolation polynomial.
  3. Missing term etimations using
  a) difference operators b) difference table
  Let us see this example

x	45	50	55	60	65
y	3	...	2	...	2.4

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- we know
- $$\Delta^3 y_0 = 0$$

  but

  $$\Delta = E - 1$$

  therefore

  $$(E-1)^3 y_0 = 1 = (E^3 - 3E^2 + 3E - 1)y_0$$

Assignment

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- $$y_3 - 3y_2 + 3y_1 - y_0 = 0 \tag{1}$$

  similarly

  $$\Delta^3 y_1 = 0$$

  gives

  $$y_4 - 3y_3 + 3y_2 - y_1 = 0 \tag{2}$$

Estimate Missing Terms

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- Solve for given values and estimate the required missing terms with the help of above two equations and the given table. Using Difference Table right