Probability And Statistics

G.N. Dar

Goals:Processes type, Definition of various terms, Probability, Conditional Probability,Law of total Probability.

June 3, 2020

Processes type : Two Types of Processes in nature

Deterministic Processes : 100% certainty

Sun rise, Sun set, day and night , Friday, Sunday etc.

Probabilistic Processes : lack 100% certainty

Layman : Qader, Eid, Ramadhan, Weather etc

Probability is not the answer but a guide and tends to be answer under conditions (will be known soon)

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a)Determinsitic Processes

Where the outcome is known in advance with surity say our physical laws

a) Ohm's law, Voltage drop is always proportional to Current $V \propto I$

b) Gas law $PV = nRT$ or $P \propto V^{-1}$

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b) Probabilistic Process

Random Experiment (RE)

Outcome of RE is not certainly defined but prediced with some Probability raniging from $0 \to 1$.

Outcome of some random experiments

a) Toss 1 or 2 coins $\Omega = \{H, T\}$ or $\Omega = \{HH, HT, TH, TT\}$

b) Throwing a die $S = \{1, 2, 3, 4, 5, 6\}$

c) Two fair dice throw, 36 possible outcomes
{11,12,13,14,15,16,21,22,23,24,25,26,31,32,33,34,35,36,41,42,43,
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Some terms and their definitions

We will understand few terms related to random experimetns or the Probabilistic Processes like

- Sample Space
- Event
- Mutually Exclusive Events
- Independent Events
- Complement Events
- New Events and Probability
- Conditional Probability
- Replacement
- without Replacement
- Total Probability
- Set of examples

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Parameters of Random experiment

Sample Space (Ω)

All the possible outcome of any randome experiment

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Parameters of Random experiment

Event E

Subset of the sample space is known as event. In a two dice throw, we have 36 possible outcomes and each outcome is an event. Tossing a coin we have two events say Head and Tail.

We can also define events as in single die having $\{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$ made of odds

$B = \{2, 4, 6\}$ made of even

$C = \{16, 25, 34, 43, 52, 61\}$ the sum is (7)

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Probability

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$$P(event) = \frac{n}{N}$$

let events A and B contain n_A and n_B cases respectively then we can define

$$P(A) = \frac{n_A}{N}, P(B) = \frac{n_B}{N}$$

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Parameters of Random experiment

Probability (contd...)

In case of coin toss (thrice)

$\Omega = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$A = \{HHH, HHT, HTH, THH, \}, \qquad P(A) = \frac{4}{8}$

$B = \{HHH, HTH, THH, TTH\}, \qquad P(B) = \frac{4}{8}$

$$(A \cap B) = \{HHH, HTH, THH\},$$

$$P(A \cap B) = \frac{3}{8}, P(A \cap B) \neq P(A).P(B)$$

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Parameters of Random experiment

Axioms (self evident)

For any event E we can see or define s

$P(E) \geq 0$

$P(\Omega) = 1$

$P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i) =$ called as axiom of additivity.

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MutuallyExclusive Events (MEE)

Two events are said to be MEE if occurance of the one prevents the occurance of other.

a) Turning left and turning right, b) Kings and Aces in cards

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Idependent Events

Two events are said to be independent if occurance of the one does not affact the occurance of other.

outcome of two dices is independent eachother.

Turning left and scratching your head can happen at the same time

Kings and Hearts

If event A and B are the independent events then we have

$$P(AB) = P(A).P(B)$$

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Complement Events

$\bar{A}$ of A consist of all events of Ω not in A.

$$(A \cup \bar{A}) = \Omega, \quad P(A \cup \bar{A}) = P(\Omega) = 1, \quad P(A \cap \bar{A}) = 0$$

In a coin toss if $A = H$ then $\bar{A} = T$, $(A \cup \bar{A}) = \{H, T\} = \Omega$

A and $\bar{A}$ are the disjoint sets or mutually Exclusive. we see

$$P(A \cup \bar{A}) = P(A) + P(\bar{A}) = P(\Omega) = 1$$

$P(\bar{A}) = P() = 1 - P(A)$

This is a useful expressions.

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New Events or Derived events from Ω

We can build up new events subsets or sets from the old ones as

$(A \cup B)$: consist of events of A, B and common of A and B

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$O/$ (empty set) for the events which does not contain any outcome.

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Examples (Assignments1)

Toss a coin 2 times

1. What is the probability

a) at most one head occur

b) at least one head occur

2. In a die throw, What is the probaibility of getting

a) P($\geq$ 4) $=$?

3. In a two die throw, find

a) $P(total = 7)$, b) of gettting double c) getting total of 10 or 11

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Assignment1 continued

Independent and MEE

4. Independent and ME event probability in tossing a coin

5 independent events occur in tossing a coin 5 times
(say ABCDE events with $P(H) = P(T) = 0.5$)

$$P(ABCDE) = P(A)P(B)P(C)P(D)P(E)$$

$$P(HTHTH) = P(HHHHH) = 0.5 * 0.5 * 0.5 * 0.5 * 0.5 = (0.5)^5 = 0.03$$

Mutually Exclusive E. zero joint probability
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$$P(HTHTH) = P(HHHHH) = 0.5 * 0.5 * 0.5 * 0.5 * 0.5 = (0.5)^5 = 0.03$$

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$$P(ABCDE) = P(A)P(B)P(C)P(D)P(E)$$

$$P(HTHTH) = P(HHHHH) = 0.5 * 0.5 * 0.5 * 0.5 * 0.5 = (0.5)^5 = 0.03$$

Mutually Exclusive E. zero joint probability
$P(AB) = 0 = P(HT) = P(TH) = 0$

Independent and MEE

4. Independent and ME event probability in tossing a coin

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(say ABCDE events with $P(H) = P(T) = 0.5$)

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Mutually Exclusive E. zero joint probability
$P(AB) = 0 = P(HT) = P(TH) = 0$

Assignment1 contd...

Picking a number between 1 and 100

A box of 100 tickets marked 1,2,3, ... , 100. A ticket is drawn at random from the box. Here are some events, with their descriptions as subsets and their probabilities obtained by counting. All possible numbers are assumed equally likely

Events	**Subsets of** $\{$**1,2,3.....,1000**$\}$	**Probability**
the number drawn has one digit	$\{1,2,...,9\}$	9%
the number drawn has two digit	$\{10,100,...99\}$	90%
the number drawn is less than or equal to the number k	$\{1,2,3....,k\}$	k%
the number drawn is strictly greater than k	$\{k,k+1,k+2,...,100\}$	(100-k)%
the sum of the digits in the number drawn is equal to 3	$\{3,12,21,30\}$	4%

Assignment1 continued

5.Two dice throw. Find the probability that the sum of two numbersis not 6 using Complement concept

If event A sum is not 6 , $\bar{A}$ is everything but 6

$$P(A) = P(\Omega) = 1 - P(\bar{A}) = 1 - \frac{5}{36}$$

$\rightarrow$	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Assignment1 continued

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Conditional Probability

A bag contains 7 red balls and 3 green balls. The probability of having a red ball in hand is going to be different than that of the probability of having green ball. Both of them is going to be different than simply having a ball in hand

Probability of an event B with a condition that an event A occurs,i,e $P(B/A)$. (new sample space or reduced or resized)

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

The probability of having a king in a pack of 52 cards is $\frac{4}{52}$ while as having a red king (condition) is just $\frac{2}{52}$

$$P(king)/P(RedKing) = P(B/A) = \frac{2}{52}$$

In this case common points are only two red kings out of 52 so $(A \cap B) = 2$ and $P(A \cap B) = \frac{2}{52}$

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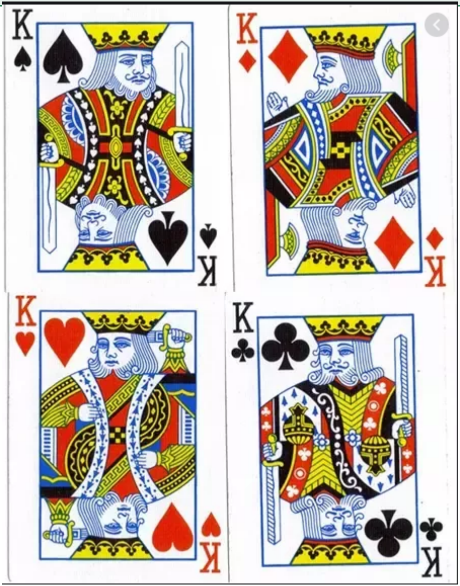
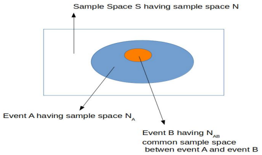


Four kings total but only two red kings.

Conditional Probability

Let $N = \Omega$, $\quad N_A = \Omega_A$, $\quad N_{AB} = \Omega_{AB}$, and $B \in A$.

$$P(A \cap B) = \frac{N_{AB}}{N} = \frac{N_{AB}}{N_A} \cdot \frac{N_A}{N}$$



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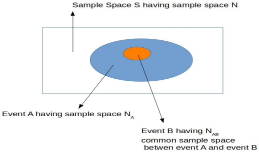

Conditional Probability

Let $N = \Omega$, $N_A = S_A$, $N_{AB} = S_{AB}$, and $B \in A$.

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Sample Space S having sample space N

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Example: Throw a fair single dice define events as $B = \{6\}$, $A = \{2, 4, 6\}$ the conditional probability $P(B/A)$ is given as

$$P(B/A) = \frac{(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

Note down the new or reduced sample space is just $\{2, 4, 6\}$

Example: for a fair single dice throw what is the probability if getting an event $A = \{1\}$ given that $B = \{1, 3, 5\}$ occured

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With and without replacement

Ex. Box conains 10 screws, 3 of which are defective. 2 screws are drawn at random. Find the probability that none of the screws are defective.

solution

a) With Replacement. Let first screw is non defective is event A

Let second screw is non defective is event B (Independent cases)

$$P(A) = \frac{7}{10}, P(B) = \frac{7}{10}$$

Probability that both the screws are non defective

$$P(A \cap B) = P(A).P(B) = \frac{7}{10}\frac{7}{10} = \frac{49}{100} = 49\%$$

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for m independent events we have

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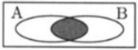
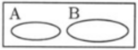
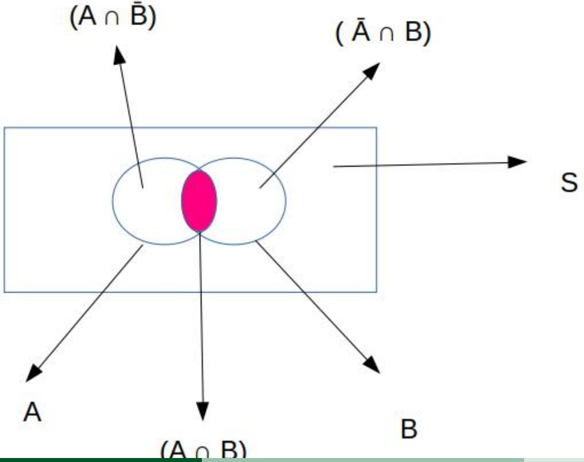
Venn diagram for various cases

Event language	Set language	Set notation	Venn diagram
outcome space	universal set	Ω	
event	subset of Ω	A, B, C, etc.	
impossible event	empty set	$\emptyset$	
not A, opposite of A	complement of A	A^c	
either A or B or both	union of A and B	$A \cup B$	
both A and B	intersection of A and B	AB, $A \cap B$	
A and B are mutually exclusive	A and B are disjoint	$AB = \emptyset$	
if A then B	A is a subset of B	$A \subseteq B$	

Additon theorem of probability: when events are not disjoint :$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



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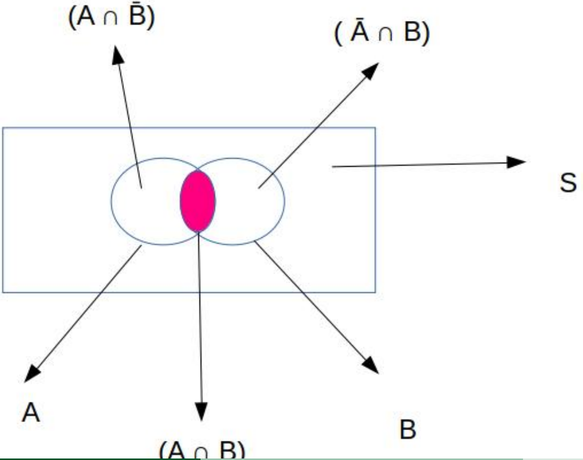

Additon theorem of probability (contd...)

from fig. $A \cup B = A \cup (\bar{A} \cap B)$

from the figure it is clear that A and $(\bar{A} \cap B)$ are disjoint. so Probability is

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Home Work: For three non mutually Exclusive events A, B, C

$P(A \cup B \cup C) = [P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) +$ P(A$\cap B \cap C)$

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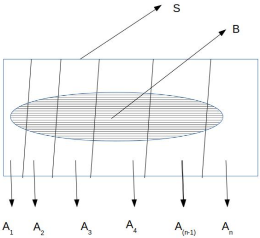
Law of Total Probability

$$P(B) = \sum_{i=1}^{n} P(B/A_i).P(A_i)$$

$$S = \Omega = \{A_1, A_2, A_3, \cdots, A_n\}$$

A_is are ME and exaust Ω

$$(A_1 \cup A_2 \cup \cdots \cup A_n) = \Omega$$



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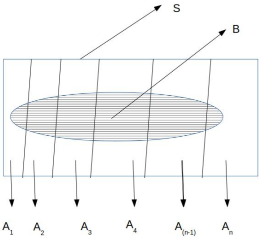

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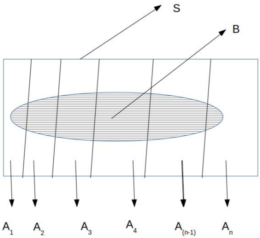

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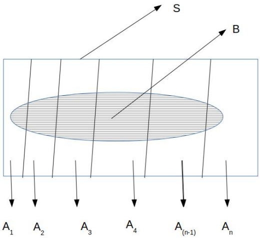

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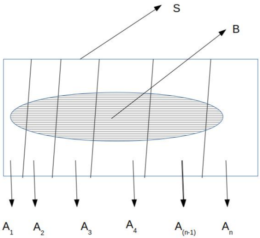

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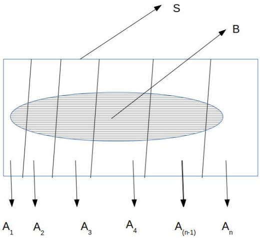

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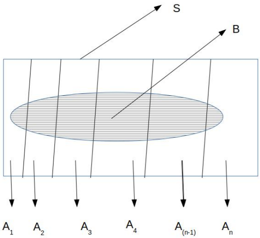

Law of Total Probability (contd...)

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(\Omega) = P(A_1) + \cdots + P(A_n) = 1$$

Let $A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup \cdots \cup A_n$

$$(A \cap B) = (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup \cdots \cup A_n) \cap B$$

from law of additionn of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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Factory X supply lamps which work $> 5000hrs$ in 99% cases while facotry Y supply lamps which work $>$ 95% cases. Facotry X produce 60% of supply and facotry Y produce 40%. What is the probability $P(lamp \geq 5000hrs)$

Solution:

$$P(A > 5000hrs) = P(A/B_X).P(B_X) + P(A/B_Y).P(B_Y)$$

$$P(A > 5000hrs) = 99\%.60\% + 95\%.40\% \ = 97.4\%$$

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Three bags, each contain 100 marbles, Bag 1 has 75 red and 25 blue marbles, Bag 2 has 60 red and 40 blue marbles; Bag 3 has 45 red and 55 blue marbles.Choose one of the bags at random and then pick a marble from the chosen bag, also at random. What is the probability that the chosen marble is red?

Solution Hint: Ball has to come from one of the bag which is equallyprobably with probability $P(B_i) = \frac{1}{3}$.

and ball from each bag varies with probability as for first $P(R/B_1) = 0.75$

then calculate total probability for selecting a red ball as per the law of total probability.

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A bucket contains 6 red 4 green balls. Two balls are drawn without replacement. Find

$$a)P(R_1), \quad b)P(R_2/R_1), \quad c)P(R_2)$$

Are the two events so drawn as R_1 and R_2 Independent. Justify.

Solution:a)

$$P(R_1) = \frac{6}{10}$$

b) (i) Directly

$$P(R_2/R_1) = \frac{5}{9}$$

(ii) this can be done also as per the conditional probability

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