Random Variables, Types and Distributions

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Outline

1. Random Variables : Dicrete
   - Probability funciton
   - Random Variables and Distribution Funciton
2. Examples
3. Continous Random variables
   - Continous Random Variables Funcitons and Distribution

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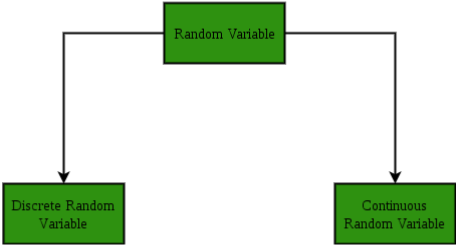
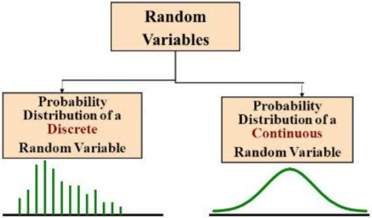
- We call it a Random Variable as it comes from a Random Experiment

Introduciton: Randome variable





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Associated to each discrete value of X say $X = x_k$ is a probability as $P(X = x_k) = f(x_k)$ called as probability function

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We define Distribution Funciton or CDF (Commulative Distribution Function) $F(x)$ as

$$F(x) = P(X \leq x), x : -\infty \leq x \leq +\infty$$

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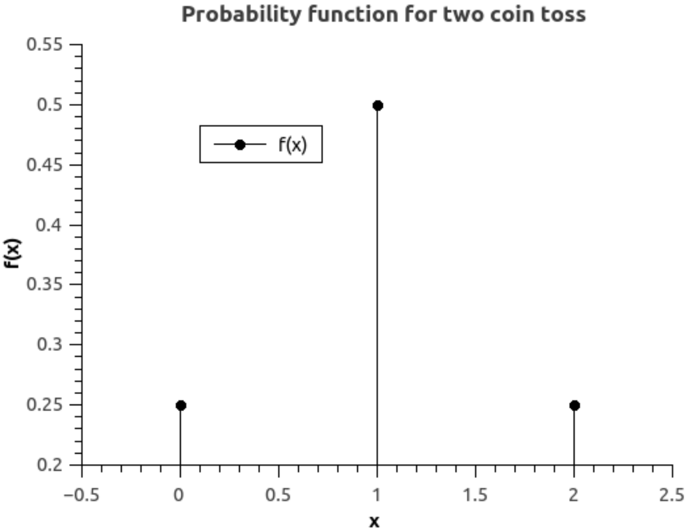

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$$P(X=1)=f(1)=\frac{1}{6}, P(X=2)=f(2)=\frac{1}{6}, P(X=3)=f(3)=\frac{1}{6}, P(X=4)=f(4)=\frac{1}{6}$$

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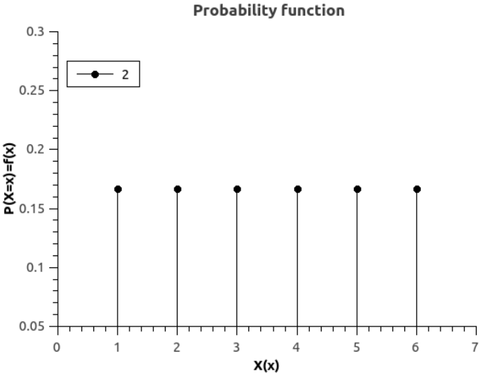

Figure: Probability function

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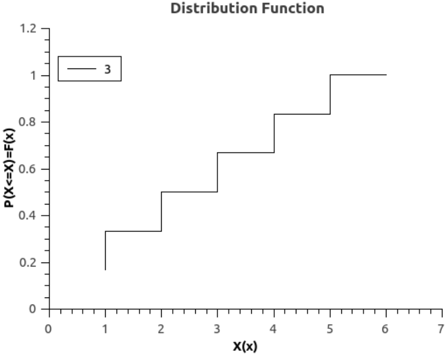

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$$\lim_{x \to +\infty} F(x) = 1$$

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The possible times that a person arrives at a restaurant.

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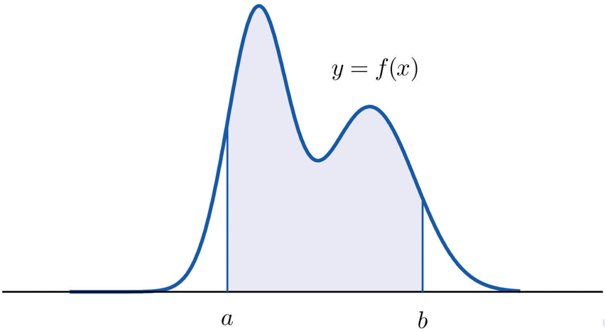
- The probability that a continuous random variable X is exactly equal to a number is zero

$P(a < X < b) = \text{area of shaded region}$



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$$= \int_{-\infty}^{-1} f(u)du + \int_{-1}^{+1} f(u)du + \int_{1}^{+\infty} f(u)du + 0 = 1$$

Example of continous functions contd..

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Similarly check

$$P(-\frac{1}{2} \leq X \leq \frac{1}{2}) = 0.6875 = 68\%$$

And

$$P(\frac{1}{4} \leq X \leq 2) = 31.64\%$$

And

$$P(X \leq x) = P(-1) = 0.95$$

$$x = 0.73$$

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- (e) Our next lecture is all about the Distributions.