8 Probing the Proton: Electron - Proton Scattering

Scattering of electrons and protons is an electromagnetic interaction. Electron beams have been used to probe the structure of the proton (and neutron) since the 1960s, with the most recent results coming from the high energy HERA electron-proton collider at DESY in Hamburg.

These experiments provide direct evidence for the composite nature of protons and neutrons, and measure the distributions of the quarks and gluons inside the nucleon.

8.1 Electron - Proton Scattering

The results of $e^-p \to e^-p$ scattering depends strongly on the wavelength $\lambda = hc/E$.

- At very low electron energies $\lambda \gg r_p$, where r_p is the radius of the proton, the scattering is equivalent to that from a point-like spin-less object.
- At low electron energies $\lambda \sim r_p$ the scattering is equivalent to that from a extended charged object.
- At high electron energies $\lambda < r_p$: the wavelength is sufficiently short to resolve sub-structure. Scattering is from constituent quarks.
- At very high electron energies $\lambda \ll r_p$: the proton appears to be a sea of quarks and gluons.

8.2 Form Factors

Extended object - like the proton - have a matter density $\rho(r)$, normalised to unit volume: $\int d^3\vec{r}\,\rho(\vec{r}) = 1$. The Fourier Transform of $\rho(r)$ is the **form factor**, $F(q)$:

$$F(\vec{q}) = \int d^3\vec{r}\,\exp\{i\vec{q}\cdot\vec{r}\}\,\rho(\vec{r}) \Rightarrow F(0) = 1 \tag{8.1}$$

Cross section from extended objects are modified by the form factor:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{extended}} \approx \left.\frac{d\sigma}{d\Omega}\right|_{\text{point-like}} |F(\vec{q})|^2 \tag{8.2}$$

For $e^-p \to e^-p$ scattering two form factors are required: F_1 to describe the distribution of the electric charge, F_2 to describe the recoil of the proton.

8.3 Elastic Electron-Proton Scattering

Elastic electron-proton scattering is illustrated in figure 8.3. As the proton is a composite object the vertex factor is modified by K^μ (compared to γ^μ):

$$\mathcal{M}(e^-p \to e^-p) = \frac{e^2}{(p_1 - p_3)^2}\,(\bar{u}_3\gamma^\mu u_1)\,(\bar{u}_4 K_\mu u_2) \tag{8.3}$$

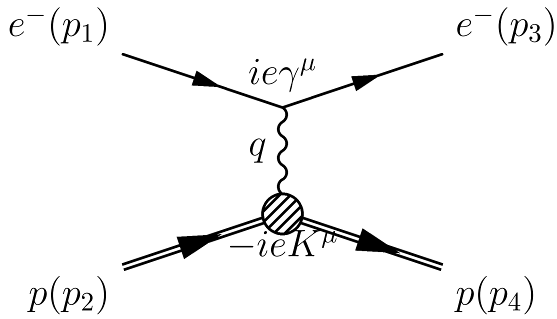


Figure 8.1: Elastic electron proton scattering. As the proton is a composite object the vertex factor is K^μ.

$$K^\mu = \gamma^\mu F_1(q^2) + \frac{i\kappa_p}{2m_p} F_2(q^2)\sigma^{\mu\nu}q_\nu \tag{8.4}$$

F_1 is the electrostatic form factor, while F_2 is associated with the recoil of the proton (as described above). F_1 and F_2 parameterise the structure of the proton, and are functions of the momentum transferred by the photon (q^2).

8.4 Elastic Scattering

The elastic scattering in the relativistic limit $p_e = E_e$ by the **Mott formula**:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} = \frac{\alpha^2}{4p_e^2 \sin^4 \frac{\theta}{2}} \left(\cos^2 \frac{\theta}{2} - \frac{q^2}{2m_p^2} \sin^2 \frac{\theta}{2}\right) \tag{8.5}$$

For θ and p_e are in the Lab frame.

In the non-relativistic limit $p_e \ll m_e$ this reduces to Rutherford scattering:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{NR}} = \frac{\alpha^2}{4m_e^2 v_e^4 \sin^4 \frac{\theta}{2}} \tag{8.6}$$

8.4.1 Q^2 and ν

We define a two new quantities: Q^2 and ν, which are useful in describing scattering.

$$Q^2 \equiv -q^2 = (p_1 - p_3)^2 > 0 \qquad \nu \equiv \frac{p_2 \cdot q}{M_p} \tag{8.7}$$

Note that $\nu > 0$ so $Q^2 > 0$ and the mass squared of the virtual photon is negative, $q^2 < 0$!

8.4.2 Higher Energy Scattering

At higher energy, we have to account for the recoil of the proton. The differential cross section for electron-proton scattering becomes:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \frac{E_3}{E_1} \left\{ \left(F_1^2 - \frac{\kappa^2 q^2}{4m_p^2} F_2^2 \right) \cos^2\frac{\theta}{2} - \frac{q^2}{2m_p^2} (F_1 + \kappa F_2)^2 \sin^2\frac{\theta}{2} \right\} \tag{8.8}$$

For a point-like spin-$\frac{1}{2}$ particle, $F_1 = 1$, $\kappa = 0$, and the above equation reduces to the Mott scattering result, equation (8.5).

It is common to use linear combinations of the form factors:

$$G_E = F_1 + \frac{\kappa q^2}{4m_p^2} F_2 \qquad G_M = F_1 + \kappa F_2 \tag{8.9}$$

which are referred to as the **electric** and **magnetic** form factors, respectively.

The differential cross section can be rewritten as:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right) \tag{8.10}$$

where we have used the abbreviation $\tau = Q^2/4m_p^2$.

The experimental data on the form factors as a function of q^2 are correspond to an exponential charge distribution:

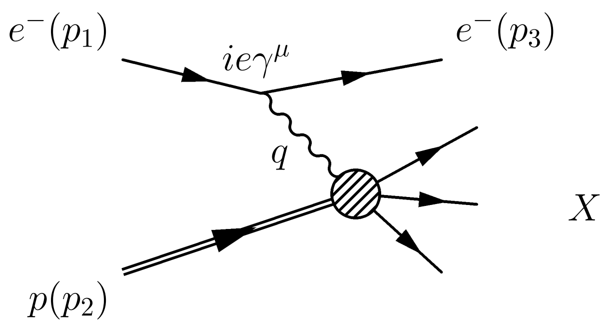
$$\rho(r) = \rho_0 \exp(-r/r_0) \qquad 1/r_0^2 = 0.71\ \text{GeV}^2 \qquad \langle r^2 \rangle = 0.81\ \text{fm}^2 \tag{8.11}$$

The proton is an extended object with an rms radius of 0.81 fm.

The whole of the above discussion can be repeated for electron-neutron scattering, with similar form factors for the neutron.

8.5 Deep Inelastic Scattering

During inelastic scattering the proton can break up into its constituent quarks which then form a **hadronic jet**. At high q^2 this is known as deep inelastic scattering (DIS).



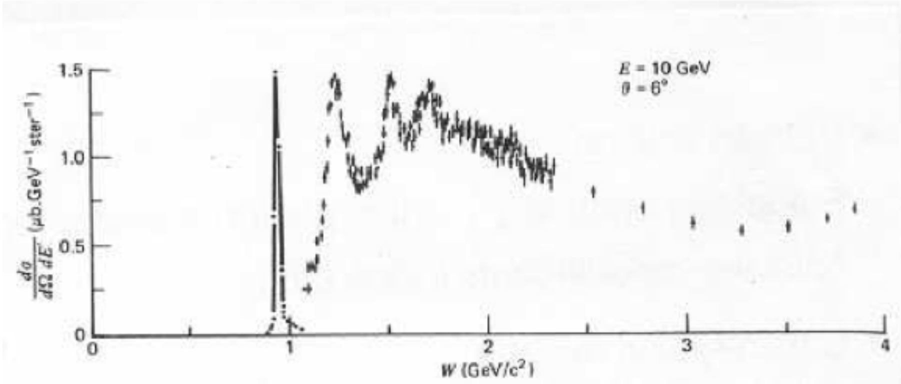


Figure 8.2: Differential cross section for E_1 =10 GeV and $\theta = 6°$, as a function of the hadronic jet mass m_X (labeled W in plot). The peaks represent elastic scattering, and then the excitation of baryonic resonances at higher mass. Above 2 GeV there is a continuum of non-resonant scattering. This data was taken by Friedman, Kendall and Taylor at SLAC in the 1960 and 1970's and was accepted as the first experimental proof of quarks. Friedman, Kendall and Taylor won the Nobel prize in 1990 for this work.

We introduce a third variable, known as Bjorken x:

$$x \equiv \frac{Q^2}{2p_2 \cdot q} \tag{8.12}$$

where again q^2 is the four-momentum transferred by the photon.

The invariant mass, M_X, of the final state hadronic jet is:

$$M_X^2 = p_4^2 = (q + p_2)^2 = m_p^2 + 2p_2 q + q^2 \tag{8.13}$$

The system X will be a baryon. The proton is the lightest baryon, therefore $M_X > m_p$.

Since $M_X \neq m_p$ q^2 and ν, are two independent variables in DIS, and it is necessary to measure E_1, E_3 and θ in the Lab frame to determine the full kinematics.

Futhermore

$$Q^2 = 2p_2 \cdot q + M_p^2 - M_X^2 \;\Rightarrow Q^2 < 2p_2 \cdot q \tag{8.14}$$

Implying the allowed range of x is 0 to 1. $0 < x < 1$ represents inelastic scattering, $x = 1$ represents elastic scattering.

8.6 The Parton Model

The parton model was proposed by Feynman in 1969, to describe deep inelastic scattering in terms of point-like constituents inside the nucleon known as **partons** with an effective mass $m < m_p$. Nowadays partons are identified as being quarks or gluons.