Chapter 13

Formulation of the Standard Model

13.1 Introductory Remarks

In Chapter 9, we discussed the properties of only a few low-mass hadrons discovered prior to the mid 1970s. As the energies of accelerators increased, additional excited states of those particles, but with larger masses and higher spins, as well as particles with new flavors (see Table 9.5) were found. In fact, even by the mid 1960s, there was a whole host of particles to contend with, and it was questioned whether they could all be regarded as fundamental constituents of matter. As we argued previously, even the lightest baryons, namely the proton and the neutron, show indirect evidence of substructure. For example, the large anomalous magnetic moments observed for these particles, especially dramatic for the neutron, imply a complex internal distribution of currents. From the pattern of the observed spectrum of hadrons, Murray Gell-Mann and George Zweig suggested independently in 1964 that all such particles could be understood as composed of quark constituents. As shown in Table 9.5, these constituents had rather unusual properties, and were initially regarded as calculational tools rather than as true physical objects.

A series of measurements performed in the late 1960s at the Stanford Linear Accelerator Center (SLAC) on electron scattering from hydrogen and deuterium revealed that the data could be most easily understood if protons and neutrons were composed of point-like objects that had charges of $-\frac{1}{3}\,e$ and $+\frac{2}{3}\,e$. These experiments, led by Jerome Friedman, Henry Kendall and Richard Taylor, corresponded to a modern parallel of the original work of the Rutherford group, where, instead of finding "point-like" nuclei within atoms, the presence of point-like *quarks* or *partons* was deduced from the characteristics of inelastically scattered electrons. In the original experiments of the Rutherford group, nuclei were not probed very deeply, and therefore did not break apart in collisions with α-particles. On the other hand, the scattering of electrons at SLAC involved sufficient momentum transfers to break apart the neutrons and protons.

It is perhaps worth expanding somewhat on the difference between elastic and inelastic scattering of electrons from nucleon targets. For elastic scattering at high energy, the form factor of Eq. (2.14), obtained from measurements at low energies, provides an adequate description of the differential cross section. However, inelastic scattering, where the proton does not stick together, offers the possibility of probing for substructure within the nucleon. In particular, the inelastic scattering of electrons at large q^2 corresponds to interactions that take place at very small distances, and are therefore sensitive to the presence of point-like constituents within the nucleon. In fact, the form factor for inelastic scattering at large q^2 becomes essentially independent of q^2, reflecting the presence of point-like objects within the nucleon. This is reminiscent of the large-angle contribution to the Rutherford scattering of low-energy α-particles on the "point-like" nucleus of the atom. It was eventually clarified that the nucleon contained charged quark-partons as well as neutral *gluon*-partons, both described by their individual characteristic momentum distributions (the *parton distribution functions*).

By the early 1970s, it became quite apparent that hadrons were not fundamental point-like objects. In contrast, leptons still do not exhibit any evidence of structure, even at highest momentum transfers. It is natural therefore to regard leptons as elementary, but to regard hadrons as composed of more fundamental constituents. This line of thought – completely phenomenological in the beginning – merged the observations from electron scattering with those from particle spectroscopy and the quark model, and culminated in the present Standard Model. The Standard Model incorporates all the known fundamental particles, namely, the quarks, leptons and the gauge bosons, and it provides a theory describing three of the basic forces of nature – the strong, weak and electromagnetic interactions.

13.2 Quarks and Leptons

As we saw earlier, each charged lepton has its own neutrino, and there are three families (or flavors) of such leptons, namely,

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}. \tag{13.1}$$

In writing this, we have used the convention introduced previously in connection with strong isospin symmetry, namely, the higher member of a given multiplet carries a higher electric charge. The quark constituents of hadrons also come in three families (see Problem **9.4**)

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} t \\ b \end{pmatrix}. \tag{13.2}$$

The charges and baryon content of the different quarks were given in Table 9.5. The baryon numbers are $B = \frac{1}{3}$ for all the quarks, and the charges are

$$\begin{aligned} Q[u] &= Q[c] = Q[t] = +\frac{2}{3}\, e, \\ Q[d] &= Q[s] = Q[b] = -\frac{1}{3}\, e. \end{aligned} \tag{13.3}$$

Although the fractional nature of their electric charges was deduced indirectly from electron scattering for only the u and d quarks, phenomenologically, such charge assignments also provide a natural way for classifying the existing hadrons as bound states of quarks. Quarks also appear to have flavor quantum numbers, as given in Table 9.5. For example, because we defined the strangeness of the K^+ as $+1$, we will see shortly that the strange quark will have to be assigned a strangeness of -1. The charm, top and the bottom quarks, correspondingly, carry their own flavor quantum numbers. Of course, each quark has its own antiquark, which has opposite electric charge and other internal quantum numbers such as strangeness and charm.

13.3 Quark Content of Mesons

The quarks, like the leptons, are point-like fermionic particles. In other words, they have spin angular momentum of $\frac{1}{2}\hbar$. This suggests that, since mesons have integer spin, then, if they are bound states of quarks, they can only consist of an even number of these particles. In fact, every known meson can be described as a bound state of a quark and an antiquark. Thus, for example, a π^+ meson, which has spin zero and electric charge $+1$, can be described as the bound state

$$\pi^+ = u\bar{d}. \tag{13.4}$$

It follows, therefore, that the π^- meson, which is the antiparticle of the π^+, can be described as the bound state

$$\pi^- = \bar{u}d. \tag{13.5}$$

The π^0 meson, which is charge neutral, can, in principle, be described as a bound state of any quark and its antiquark. However, other considerations, such as the fact that all three π-mesons belong to a strong-isospin multiplet, and should therefore have the same internal structure, lead to a description of the π^0 meson as

$$\pi^0 = \frac{1}{\sqrt{2}}\,(u\bar{u} - d\bar{d}). \tag{13.6}$$

The strange mesons can similarly be described as bound states of a quark and an antiquark, where one of the constituents is strange. Thus, we can identify the following systems

$$\begin{aligned} K^+ &= u\bar{s}, \\ K^- &= \bar{u}s, \\ K^0 &= d\bar{s}, \\ \overline{K^0} &= \bar{d}s. \end{aligned} \tag{13.7}$$

It is quite easy to check that not only are the charge assignments right, but even the strangeness quantum numbers work out to be correct if we assign a strangeness quantum number $S = -1$ to the s-quark. Because there are quarks with higher mass and new flavor quantum numbers, phenomenologically, on the basis of the quark model, we would also expect new kinds of mesons. Many such mesons have already been found. For example, the charge-neutral J/ψ meson, whose discovery in 1974 by independent groups headed by Samuel Ting and by Burton Richter suggested first evidence for the existence of the charm quark, can be described as a bound state of charmonium (named in analogy with positronium)

$$J/\psi = c\bar{c}. \tag{13.8}$$

This is a "normal" meson, in the sense that the quantum numbers of charm add up to zero, but its properties (decays) cannot be explained using only the older u, d and s quarks. There are, of course, mesons that contain *open* charm, such as

$$\begin{aligned} D^+ &= c\bar{d},\\ D^- &= \bar{c}d,\\ D^0 &= c\bar{u},\\ \overline{D^0} &= \bar{c}u. \end{aligned} \tag{13.9}$$

We can think of such mesons as the charm analogs of the K mesons, and the properties of these mesons have by now been studied in great detail. In analogy with the K^+, the D^+ meson is defined to have charm flavor of $+1$, which then defines the charm quantum number for the c-quark to be $+1$. There are also mesons that carry both strangeness and charm quantum numbers, two of these are denoted as

$$\begin{aligned} D_s^+ &= c\bar{s},\\ D_s^- &= \bar{c}s. \end{aligned} \tag{13.10}$$

Finally, there is also extensive evidence for hadrons in which one of the constituents is a bottom quark. For example, the B mesons, analogous to the K mesons, have structure of the form

$$\begin{aligned} B^+ &= u\bar{b},\\ B^- &= \bar{u}b,\\ B_d^0 &= d\bar{b},\\ \overline{B_d^0} &= \bar{d}b. \end{aligned} \tag{13.11}$$

The charge-neutral states involving b and s quarks are particularly interesting because, just like the K^0-$\overline{K^0}$ system, they exhibit CP violation in their decays

$$\begin{aligned} B_s^0 &= s\bar{b}, \\ \overline{B_s^0} &= \bar{s}b. \end{aligned} \tag{13.12}$$

Recent experiments at two e^+e^- colliders, one at SLAC ("BaBar") and one at the KEK accelerator at Tsukuba, Japan ("BELLE"), both referred to as "B-factories", have studied these neutral B mesons in the clean environment of e^+e^- collisions, and have found clear evidence for large violation of CP symmetry in decays of B^0 mesons produced in pairs

$$e^+ + e^- \longrightarrow B + \overline{B}. \tag{13.13}$$

Extensive studies of B decays are currently being pursued at both e^+e^- and at hadron colliders, to search for any discrepancies with expectations from the Standard Model.

13.4 Quark Content of Baryons

Just as mesons can be thought of as bound states of quarks and antiquarks, so can baryons be considered as constructed out of these constituents. But because baryons carry half-integral spin angular momenta (they are fermions), they can be formed from only an odd number of quarks. Properties of baryons are most consistent with being composed of only three quarks. Thus, we can think of the proton and the neutron as corresponding to the bound states

$$\begin{aligned} p &= uud, \\ n &= udd. \end{aligned} \tag{13.14}$$

Similarly, the hyperons, which carry a strangeness quantum number, can be described by

$$
\begin{aligned}
\Lambda^0 &= uds, \\
\Sigma^+ &= uus, \\
\Sigma^0 &= uds, \\
\Sigma^- &= dds. \qquad (13.15)
\end{aligned}
$$

Also, the cascade particles, which carry two units of strangeness, can be described as

$$
\begin{aligned}
\Xi^0 &= uss, \\
\Xi^- &= dss. \qquad (13.16)
\end{aligned}
$$

Since all baryons have baryon number of unity, it follows therefore that each quark must carry a baryon number of $\frac{1}{3}$. Furthermore, since a meson consists of a quark and an antiquark, and since an antiquark would have a baryon number $-\frac{1}{3}$, we conclude that mesons do not carry baryon number, which is consistent with our previous discussion.

13.5 Need for Color

Extending the quark model to all baryons, leads to a theoretical difficulty. We have already discussed the Δ^{++} baryon, which is nonstrange, carries two units of positive charge, and has spin angular momentum of $\frac{3}{2}$. Thus, naively, we can conclude that the Δ^{++} can be described by three up quarks

$$\Delta^{++} = uuu. \qquad (13.17)$$

This substructure satisfies all the known quantum numbers, and, in the ground state (where there are no contributions from relative orbital waves), the three up quarks can have parallel spins to provide a resultant value of $J = \frac{3}{2}$. However, the wave function for this final state, representing three identical fermions, would therefore be symmetric under the exchange of any two quarks. This is, of course, incompatible with the Pauli principle, which requires a wave function containing identical fermions to be totally antisymmetric. It would appear, therefore, that the quark model cannot describe the Δ^{++}. On the other hand, the model works so well for other hadrons that it would seem unwise to give it up entirely. An interesting resolution can be attained if it is assumed that all quarks carry an additional internal quantum number, and that the final state in Eq. (13.17) is, in fact, antisymmetric in the space corresponding to this quantum number.

This additional degree of freedom is referred to as *color*, and it is believed that each of the quarks comes in three different colors. Namely, the quark multiplets take the form

$$\begin{pmatrix} u^a \\ d^a \end{pmatrix}, \quad \begin{pmatrix} c^a \\ s^a \end{pmatrix}, \quad \begin{pmatrix} t^a \\ b^a \end{pmatrix}, \qquad a = \text{ red, blue, green.} \tag{13.18}$$

At this point of our development, color can be regarded as merely a new quantum number needed for phenomenological reasons for understanding the substructure of hadrons. However, we will see shortly that, in fact, color is to the strong interaction what charge is to the electromagnetic force, namely the source of the respective fields.

Hadrons do not appear to carry any net color, and therefore correspond to bound states of quarks and antiquarks of zero total color quantum number, or, simply stated, hadrons are color-neutral bound states of quarks. Under the interchange of any two quarks, the color singlet wave function of three quarks changes sign, while that of a quark-antiquark color singlet does not. This hypothesis leads to an excellent description of all known baryons as bound states of three quarks, and of mesons as bound states of quark-antiquark pairs. In particular, it also explains the structure of the Ω^- baryon which has a strangeness of -3 and spin angular momentum of $\frac{3}{2}$, and corresponds to the ground state of three strange quarks

$$\Omega^- = sss. \tag{13.19}$$

We see once again how the symmetry property in color space plays a crucial role in assuring the overall antisymmetry of the fermionic wave function for this state.

The theoretical postulate of color seems rather ad hoc, especially since the observable hadrons do not carry a color quantum number. However, the existence of color can be established as follows. Consider the annihilation of an electron and positron, leading to the creation of a $\mu^+\mu^-$ pair or a quark-antiquark pair. The reaction can be thought of as proceeding through the production of an intermediate virtual photon, as shown in Fig. 13.1. The cross section for the production of hadrons in this process depends on the number of ways a photon can produce a quark-antiquark pair. This must therefore be proportional to the number of available quark colors. That is, the ratio of production cross sections

$$R = \frac{\sigma(e^- e^+ \to \text{hadrons})}{\sigma(e^- e^+ \to \mu^- \mu^+)}, \tag{13.20}$$

is proportional to the number of quark colors. And, indeed, this quantity is consistent with exactly three colors. Because the production of hadrons through the mechanism in Fig 13.1 depends, in addition, on the electric charges of the quarks, such data also confirm the fractional nature of electric charge carried by quarks.

In closing this section, we wish to point out that electron-positron annihilation at high energies is one of the cleanest ways to establish the presence of new quark flavors. For example, when the energy of the e^+e^- system exceeds the threshold for production of hadrons containing some new quark, the ratio in Eq. (13.20) must increase and display a step at that energy. Of course, in addition, beyond that threshold, new hadrons containing the new flavor can be observed in the final states of such collisions. This was found to be the case for charm and bottom quarks, where after the initial production of the analogs of positronium (the J/ψ for $c\bar{c}$, and the Υ for $b\bar{b}$), particles with open charm and open bottom flavor were observed to be produced at somewhat higher energies. However, this is not likely to repeat for the top quark, which, as we have indicated before, decays too rapidly after its production to be able to form hadrons.

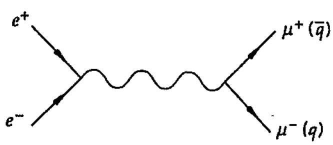

Fig. 13.1 The annihilation of e^+e^- through a virtual photon into $\mu^+\mu^-$ or $q\bar{q}$ pair.

13.6 Quark Model for Mesons

We will now apply the symmetry requirements of the strong interaction to $q\bar{q}$ wave functions, and thereby deduce the quantum numbers that we would expect for the spectrum of charge-neutral meson states in a simple non-relativistic quark model. Specifically, we will establish the restrictions on the spin (J), parity (P) and charge conjugation (C) quantum numbers that apply to such systems. The $q\bar{q}$ wave function is a product of separate wave functions, each of which has a unique symmetry under the exchange of the two particles

$$\Psi = \psi_{\text{space}}\ \psi_{\text{spin}}\ \psi_{\text{charge}}, \tag{13.21}$$

where ψ_{space} denotes the space-time part of the $q\bar{q}$ wave function, ψ_{spin} represents the intrinsic spin, and ψ_{charge} the charge conjugation properties. We have ignored the part of the wave function that is associated with the color degree of freedom because we know that this will always have even symmetry for mesons.

The symmetry of ψ_{space} under the exchange of the q and $\bar{q}$ is, as usual, determined by the spherical harmonics and the relative orbital angular momentum of the q and $\bar{q}$. If we call the exchange operation X, then, schematically, we have

$$X\psi_{\text{space}} \sim X\ Y_{\ell m}(\theta,\phi) = (-1)^{\ell}\ \psi_{\text{space}}. \tag{13.22}$$

Therefore, if Ψ is a state of definite parity, the spatial part of the wave function will be either symmetric or antisymmetric under exchange, depending on whether ℓ is even or odd.

The effect of the exchange operation on ψ_{spin} will depend on whether the two quark spins are in a spin state $s = 0$ or $s = 1$. Considering the states with $s_z = 0$, we obtain

$$\begin{aligned} s = 0: &\quad X[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle] = -[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle], \\ s = 1: &\quad X[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle] = +[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle]. \end{aligned} \tag{13.23}$$

Thus, we deduce that

$$X\psi_{\text{spin}} = (-1)^{s+1}\psi_{\text{spin}}. \tag{13.24}$$

Under the action of the exchange operator, q and $\bar{q}$ become interchanged, and consequently, we can think of this as the operation of charge conjugation in the space of ψ_{charge}. To determine the charge conjugation properties of such a state, let us impose the Pauli principle on our two-fermion system, namely, let us require that the overall wave function change sign under an interchange of q and $\bar{q}$. Note that we are using a generalized form of the Pauli principle, which treats q and $\bar{q}$ as identical fermions corresponding to spin up and spin down states in the space of ψ_{charge}. Thus, we require

$$X\Psi = -\Psi. \tag{13.25}$$

Now, using the results of Eqs. (13.22), (13.24) and (13.25), we can write

$$\begin{aligned} X\Psi &= X\psi_{\text{space}} X\psi_{\text{spin}} X\psi_{\text{charge}} \\ &= (-1)^{\ell}\psi_{\text{space}}(-1)^{s+1}\psi_{\text{spin}} C\psi_{\text{charge}} = -\Psi. \end{aligned} \tag{13.26}$$

Consequently, for Eq. (13.26) to hold, we conclude that the meson state must be an eigenstate of charge conjugation with charge parity

$$\eta_C = (-1)^{\ell+s}. \tag{13.27}$$

Thus, for meson states that are eigenstates of charge conjugation, Eq. (13.27) establishes a relationship between the orbital wave, the intrinsic-spin value, and the C quantum number of the $q\bar{q}$ system.

The only relevant quantum number that is still missing in our discussion is the parity of the allowed states. The parity of Ψ is given by the product of the intrinsic parities of the constituents and the effect from inversion of spatial coordinates. As we discussed in Chapter 11, the relative intrinsic parity of a particle and an antiparticle with spin $\frac{1}{2}$ is odd. Consequently, the total parity of our state Ψ is

$$P\Psi = -(-1)^{\ell}\Psi = (-1)^{\ell+1}\Psi,$$

or, the total parity quantum number is

$$\eta_P = (-1)^{\ell+1}. \tag{13.28}$$

Since the spins of mesons are obtained from the addition of the orbital and intrinsic angular momenta of the $q\bar{q}$ pair

$$\vec{J} = \vec{L} + \vec{S}, \tag{13.29}$$

we now have all the ingredients for forming an allowed spectrum of mesons. Table 13.1 lists the possible lowest-lying states, all of which correspond to known mesons.

Table 13.1 Lowest-lying meson states expected in the quark model.

ℓ	s	j	η_P	η_C	*Meson*[a]
0	0	0	$-$	$+$	π^0, η
0	1	1	$-$	$-$	ρ^0, ω, ϕ
1	0	1	$+$	$-$	$b_1^0(1235)$
1	1	0	$+$	$+$	$a_0(1980), f_0(975)$
1	1	1	$+$	$+$	$a_1^0(1260), f_1(1285)$
1	1	2	$+$	$+$	$a_2^0(1320), f_2(1270)$

[a]For other properties of these mesons, see the *CRC Handbook.*

13.7 Valence and Sea Quarks in Hadrons

Regarding the substructure of hadrons, we have mentioned that the nucleon appears to contain both quarks (q) and gluons (g), and, in fact, we have also mentioned a momentum distribution for partons. The quark model of hadrons must be therefore be recognized as the analog of the description of valence electrons in an atom or the valence nucleons in a nucleus. Just as there are more constituents in the closed shells of atoms or nuclei, so there are also more "paired" quark-antiquark systems within hadrons. These quarks are referred to as the *sea quarks*, as opposed to the *valence quarks* that characterize hadronic quantum numbers. And, in fact, the additional contribution from the color-carrying gluons (see following two sections) corresponds to about half of the content of the nucleon.

Searches for other kinds of hadrons have been performed in order to identify new possible states of matter that would correspond to, for example, valence systems of $q\bar{q}q\bar{q}$, *hybrid* mesons composed of $q\bar{q}g$, or *glueballs* made of gg systems. There is some evidence for the existence of such expected states, but it is as yet not compelling.

13.8 Weak Isospin and Color Symmetry

As we have seen, leptons and quarks come as doublets, or in pairs, and quarks, in addition, carry a color quantum number. The existence of such groupings, and the color degrees of freedom, suggest the presence of new underlying symmetries for this overall structure. From our discussion of spin and isospin, we can associate the doublet structure with a non-commuting (non-Abelian) symmetry group $SU(2)$. We will continue to refer to this underlying symmetry group as isospin, since it is an internal symmetry. Unlike strong isospin, which is used only to classify hadrons, the isospin in the present case also classifies leptons. Leptons, on the other hand, interact weakly and, therefore, this symmetry must be related to the weak interaction. Correspondingly, the isospin symmetry associated with the weak interactions of quarks and leptons is referred to as *weak isospin*. This symmetry is quite distinct from that of the strong isospin symmetry that we discussed previously. But, as with strong isospin, where the symmetry is discernible only when the electromagnetic interaction (electric charge) can be ignored, so is the essential character of the weak isospin symmetry also apparent only when the electromagnetic force is "turned off". Under such circumstances, the up and down states of Eqs. (13.1) and (13.2) become equivalent and cannot be distinguished.

For the case of weak-isospin symmetry, we can define a weak hypercharge for each quark and lepton, based on a general form of the Gell-Mann–Nishijima relation of Eq. (9.26), namely,

$$\begin{aligned} Q &= I_3 + \frac{Y}{2}, \\ \text{or} \quad Y &= 2(Q - I_3), \end{aligned} \tag{13.30}$$

where Q is the charge of the particle, and I_3 the projection of its weak isospin quantum number. Thus, for the (ν, e^-) doublet we obtain

$$Y(\nu) = 2\left(0 - \frac{1}{2}\right) = -1,$$

$$Y(e^-) = 2\left(-1 + \frac{1}{2}\right) = -1. \tag{13.31}$$

Similarly, for the (u, d) quark doublet, we have

$$Y(u) = 2\left(\frac{2}{3} - \frac{1}{2}\right) = 2 \times \frac{1}{6} = \frac{1}{3},$$

$$Y(d) = 2\left(-\frac{1}{3} + \frac{1}{2}\right) = 2 \times \frac{1}{6} = \frac{1}{3}. \tag{13.32}$$

The weak hypercharge quantum number for other quark and lepton doublets can be obtained in the same manner. In fact, in the Standard Model, only left-handed particles have a doublet structure. The right-handed quarks and the right-handed charged leptons are all singlets with $I = 0$, and there are no right-handed neutrinos. As can be seen from Eq. (13.30), the weak hypercharge quantum number is the same for both members of any doublet, which is required if weak hypercharge is to be regarded as a $U(1)$ symmetry of the type specified in Eq (10.76).

The color symmetry of quarks is also an internal symmetry. It can be shown that it is similar to isospin in that it involves rotations – however, the rotations are in an internal space of three dimensions – corresponding to the three distinct colors of the quarks. The relevant symmetry group is known as $SU(3)$. The interactions of quarks are assumed to be invariant under such $SU(3)$ rotations in color space, leading to an equivalence of quarks of different color. (This is needed in order to have consistency with experimental observations.) Because the color quantum number is carried by quarks and not by leptons or photons, we expect this symmetry to be associated only with the strong interaction.

13.9 Gauge Bosons

As we have seen, the presence of a global symmetry can be used to classify particle states according to some quantum number (e.g., strong isospin), while the presence of a local symmetry requires the introduction of forces. Since weak isospin and color symmetry are associated with rather distinct interactions, it is interesting to ask whether the corresponding physical forces – namely, the strong (color) and the weak forces – might arise purely from the requirement that these symmetries be local. Years of painstaking theoretical development, coupled with detailed experimental verification, has led to the conclusion that this is indeed very likely. It is the current understanding that the local symmetries underlying the electromagnetic, weak, and strong interactions have origin in the $U_Y(1)$, $SU_L(2)$, and $SU_{\text{color}}(3)$ symmetry groups, respectively. The group corresponding to the weak hypercharge symmetry, $U_Y(1)$, is a local Abelian symmetry group, while $SU_L(2)$ and $SU_{\text{color}}(3)$ are non-Abelian groups corresponding to the weak isospin and color symmetries.[1] From the Gell-Mann–Nishijima formula of Eq. (13.30), we see that electric charge is related to weak hypercharge and weak isospin, from which it follows that the electromagnetic $U_Q(1)$ symmetry can be regarded as a particular combination of the weak isospin and weak hypercharge symmetries.

In Chapter 10, we showed in an example how local invariance necessarily leads to the introduction of gauge potentials, such as the vector potential in electromagnetic interactions. When these potentials are quantized, they provide the carriers of the force, otherwise known as gauge particles. Thus, the photon is the carrier of the electromagnetic interaction, or its gauge boson. All the gauge bosons have spin $J = 1$, and the number of gauge bosons associated with any symmetry reflects the nature of that symmetry group. There are three gauge bosons associated with the weak interactions, and they are known as the W^+, W^-, and Z^0 bosons. (These were discovered independently in 1983 by Carlo Rubbia and collaborators and Pierre Darriulat and collaborators at the antiproton-proton collider at the CERN Laboratory outside of Geneva, Switzerland.) For the strong interactions, there are eight gauge bosons, and all are referred to as gluons. (These are the same gluons we have been discussing in connection with the substructure of the nucleon.) The gluons, or the gauge bosons of color symmetry, are electrically neutral, but carry the color quantum number. This is in contrast to the photon, which is the carrier of the force between charged particles, but does not itself carry electric charge. This difference can be attributed to the Abelian nature of the $U_Q(1)$ symmetry that describes the photon, and the non-Abelian nature of $SU_{\text{color}}(3)$ that describes gluons.

[1] Because the doublet structure of quarks and leptons involves only left-handed particles, the weak isospin symmetry group is also conventionally denoted by $SU_L(2)$. This kind of structure is essential for incorporating the properties of neutrinos and of parity violation in weak interactions.