Binomial Distribution

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Outline

1. Binomial Distribution
2. Binomial Distribution
   - Derivation of Binomial Distribution
   - Example
3. Parameters of Binomial Distribution
   - Expectation Value
   - Variance

Introduction

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- Number of distributions, discrete and continous (due to discrete and continous RV).
- Binomial and Bernouli Distributions and its parameters like expectation valueand variance

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- X={ 0,1,2,3,4,......n}
- This will follow a distribution called as *Binomial Distribution B(n,x)*.

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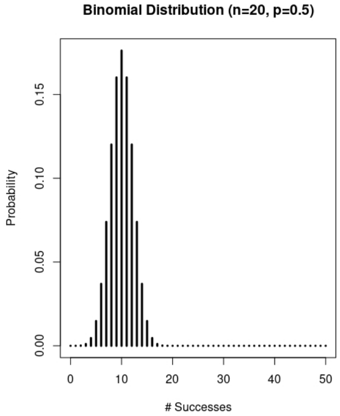

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- If r times event A occurs then $n - r$ times event B occurs (Keep in mind independent events) and one way(combination) of such possibility is

$$AAAAAAAAA \cdots A_r.BBBBBBBB \cdots B_{n-r}$$

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- This $f(x)$ follows a distribution called Binomial Distributin

$$B(n, p) =^n c_r p^r (1-p)^{n-r}$$

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Note that a die has 6 sides but here we look at only two cases: "four: yes" or "four: no"

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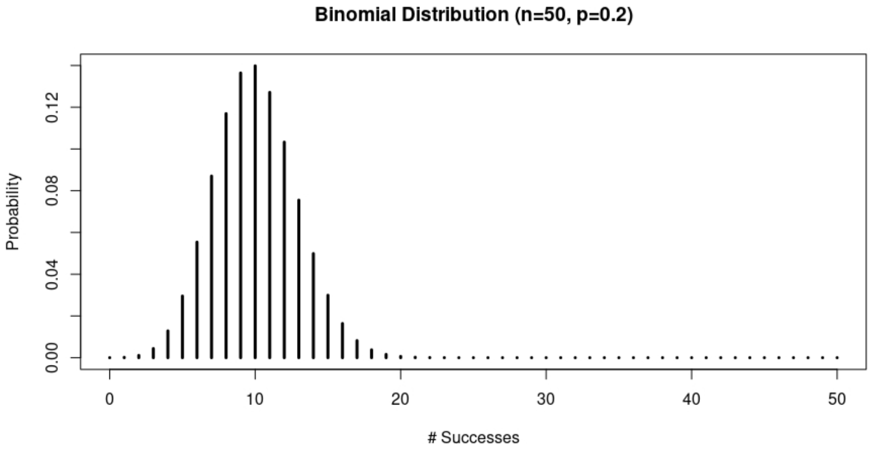

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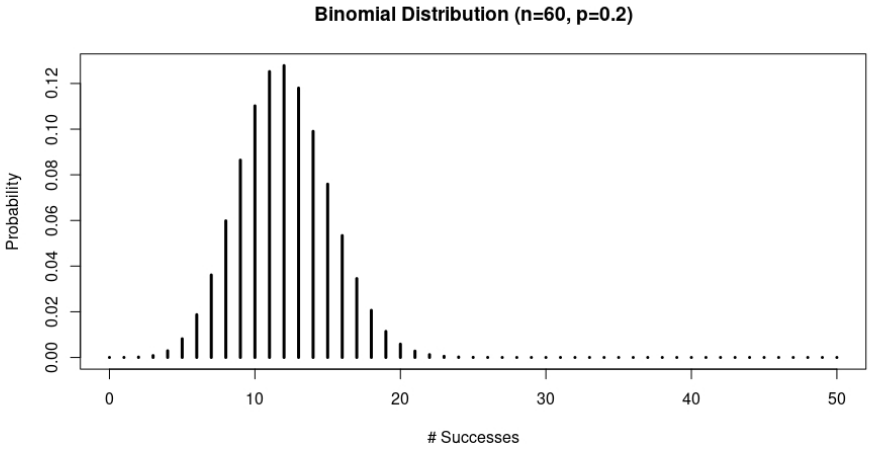

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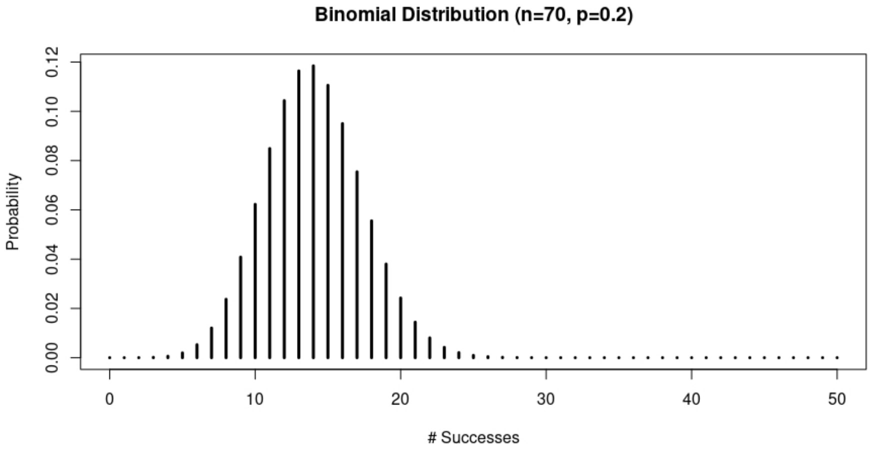

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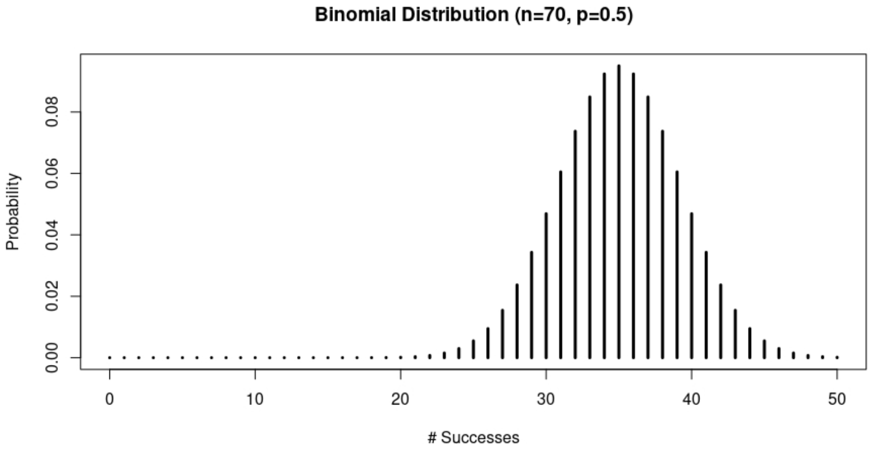

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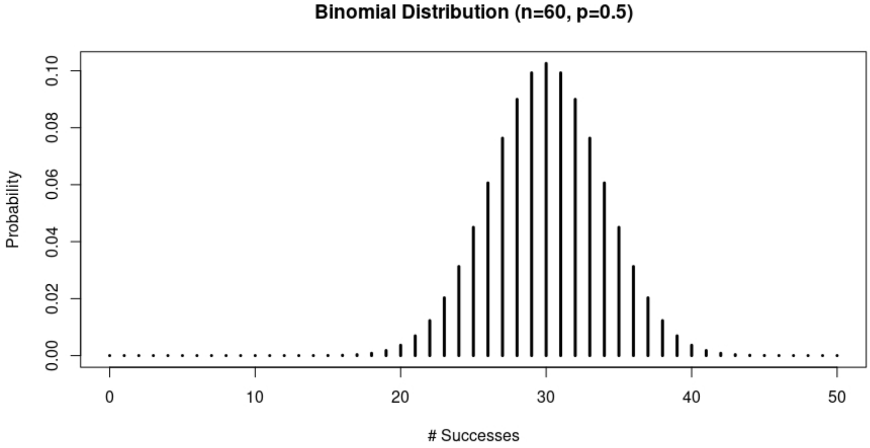

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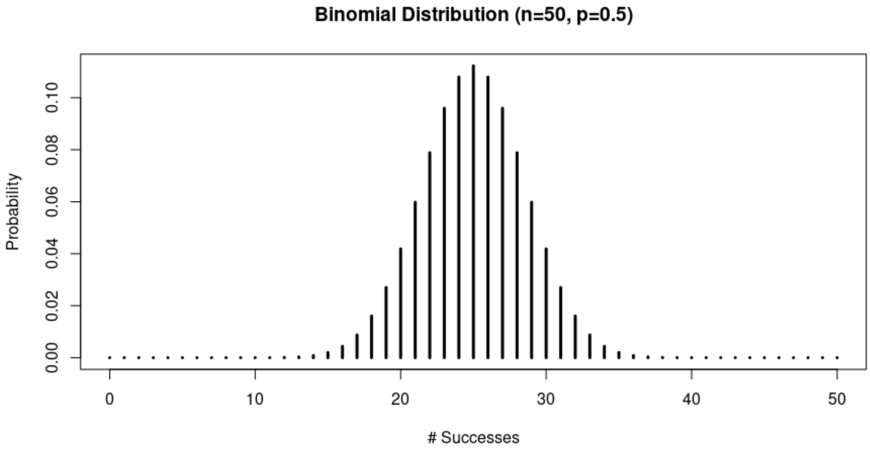

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Example contd...

- $$f(x) = \frac{1}{36}, \frac{2}{36}, \frac{6}{36} \frac{5}{.} \ldots\ldots . \frac{2}{36}, \frac{1}{36}$$

- $$f(x) = 0.03, 0.05, 0.14, \ldots..0.06, 0.03$$

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- $$^{n}c_r p^r q^{n-r} =^{n} c_r \left(\frac{1}{36}\right)^r \left(1 - \frac{1}{36}\right)^{n-r}$$

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- $$=$$

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- $${}^{n}c_r p^r q^{n-r} =^{n} c_r \left(\frac{1}{36}\right)^r \left(1 - \frac{1}{36}\right)^{n-r}$$

- $$= {}^{12}c_0 \frac{1}{36}^0 (1 - \frac{1}{36})^{12-0}$$

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- $$= {}^{12}c_0 \frac{1}{36}^0 (1 - \frac{1}{36})^{12-0} + {}^{12}c_1 \frac{1}{36}^1 (1 - \frac{1}{36})^{12-1}$$

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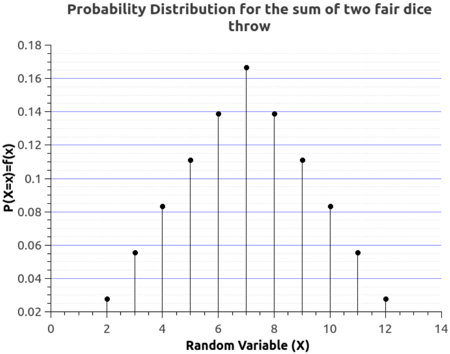

Expectation Value

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  And

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- when

  $$y = x - 1$$

Expectation value contd...

we get

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And at the same time we have some changes in index n as when

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when

$$x = 1 \rightarrow n$$

we have for y as

$$y = 0 \rightarrow (n - 1) = m$$

$$n = m + 1$$

Expectation contd..

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- Therfore

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Expectation contd..

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- Expectation value for Binomial Distribution is np

Expectation value contd...

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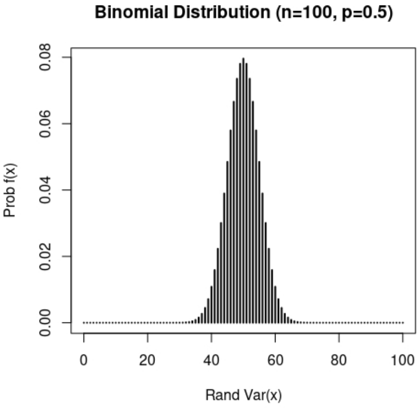

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Varaince or Standard Deviation

$$\sigma^2(x) = \nu_2$$

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