Chi Squre Distribution

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Outline

1. Chi Squre Distribution
    - Randome variable in Chi Squre Distribution
    - Applications of Chi Squre Distribution
    - Chi-squre test of goodness of fit
    - Derivation of Chi Squre distribtion for one degre of freedom
    - Expected value of Chi Squre Distribution
    - Variance of Chi Squre Distribution

Chi Squre Distribution

- Before we start with chi-squre disrtibution directly
- we are going to understand the background of it
- This distibution is used to check the theoritical hypothesis against the experimental result
- conclusions are drawn wether the random experiment is unbiased
- or the biased one
- so that acceptance or rejection of the experimental observation is done.

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- In this distribtion the important parameter is degree of freedom (df)
- say for example
- in a fair dice throw it is 5
- in coin toss it is 1 and soon
- By df, we mean independency
- for example throwing a dice 100 times ,5 outcomes say $1, 2, 3, 4, 5$ are independent
- but 6th will be dependent

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- then 6 th will be restricted to 20times only
- Similarly in tossing a coin 100 times if head appears 55 times
- then tail has to appear only 45 times and soon.
- We define degree of freedom as
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Randome variable in Chi Squre Distribution

- If $X_1, X_2, X_3, X_4, X_5, ...X_n$ are the random variables
- each of them follows a normal distibution or standard normal distribtions
- then the variable χ^2 defined as below is following a Chi-squrre distribtion.
- $$\chi^2 = Y = X_1^2 + X_2^2 + X_3^2 + X_4^2 + ... + X_n^2$$
- support base of Y
- $$R_y \in [0, \infty]$$
- $$\chi^2(y) = \begin{cases} Cy^{\frac{n}{2}-1)} e^{\frac{-1}{2}y}, & \text{if } \chi^2 \in R_x \\ 0, & \text{otherwise} \end{cases}$$

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- following a normal distribtion then the sum of squre of each of these variables follows a Chi-squre distibution.
- i)

$$\chi^2 \geq 0$$

- ii) Chi-squre is a skewed distibution for lower degree of freedom and peaks immedielty near zero
- but for larger degree of freedom > 90, it is approaching to normal distribtion with mean at df and varaince $\sqrt{2df}$

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- It has a large number of Applications and some are as
- i) Chi-squre test of goodness of fit
- ii) Chi-squre test for independence of attributes
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- It is a tool or test to describe the magnitude of discrepancy between theory and observation
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- If $O_i(i = 1, 2, 3...n)$ is observed data and $E_i(i = 1, 2, 3, 4..n)$ are the corresponding expected data (theoritical frequncy) then the statistic χ^2 defined as
- $$\chi^2 = \sum_{i=1}^{n} \left[\frac{(O_i - E_i)^2}{E_i} \right]$$
  with
- $$\sum_{i=1}^{n} O_i = \sum_{i=1}^{n} E_i$$
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- *Example*: A die thrown 132 times and results are
- Number turned up:123456 with frequncy as frequncy : 162025142928 respectively. test the hypothesis that the die is unbiased.
- solution based on hypothesis the expected: $\frac{132}{6} = 22$

O (Observed)	E (Expected)	$(O\text{-}E)^2$	$\frac{(O-E)^2}{E}$
16	22	36	1.64
20	22	4	0.18
25	22	9	0.41
14	22	64	2.91
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28	22	36	1.64
			$\sum \frac{(O-E)^2}{E} = 9.01$

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- Let z is a standard normal variable with mean 0 and varaince 1 that is $z\ S(0,1)$. Then
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- for x we know about the distribtion funciton
  $$x < 0, F_x(x) = 0$$
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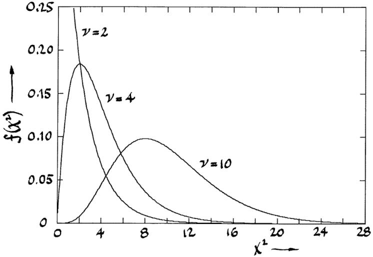
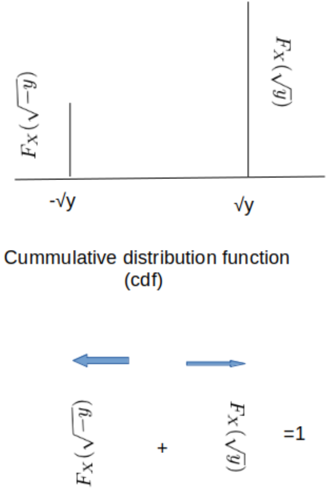


Figure 1 — The chi-square distribution for $\nu = 2$, 4, and 10.



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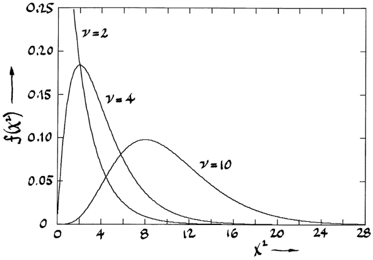

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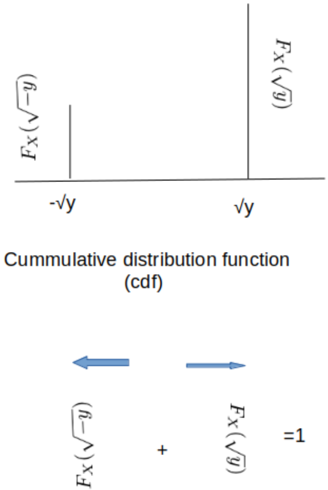

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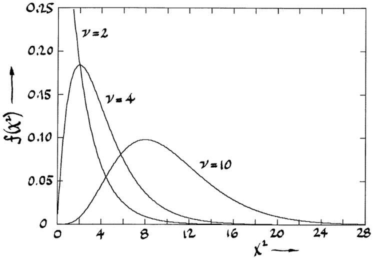

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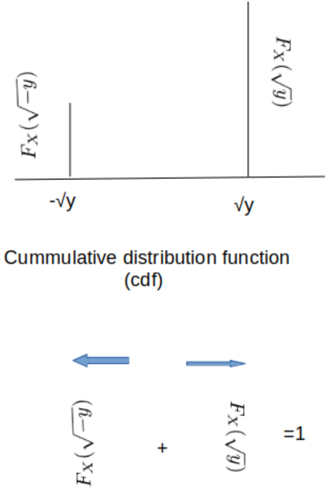

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- **Alternative Method**
- X follows standar normal distibution with mean 0 and varaince 1
- $$XSN(0,1)$$
- then the pdf for X^2 will be derived as
- $$f(x) = \frac{d}{dx}P(X^2 < x)$$
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- in standard notaiton we write

$$\chi^2_x(x) = \frac{1}{2^{\frac{1}{2}}\Gamma(\frac{1}{2})} x^{(\frac{1}{2}-1)} e^{-(\frac{1}{2}x)}$$

- If x follows a χ^2 distribtion then

$$E(X) = \int_0^\infty x\chi^2(x)dx$$

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Expected value of Chi Squre Distribution

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- $$E(X) = C\int_0^\infty x^{(\frac{n}{2})}e^{(-\frac{1}{2}x)}dx$$
- $$E(X) = C[-x^{\frac{n}{2}}2e^{(-\frac{1}{2}x)}]_0^\infty + C\int_0^\infty \frac{n}{2}x^{(\frac{n}{2}-1)}2e^{(-\frac{1}{2}x)}dx$$
- do some calculations and see we get back a distribtion which gives probability from all over the support base as 1

$$= n\int_0^\infty \chi(x)dx = n.1 = n$$

hence expected value is the degree of freedom

Expected value of Chi Squre Distribution

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- we know

$$\nu_2 = V(X) = E(X^2) - (E(X))^2 = E(X^2) - \mu^2$$

therefore

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Variance of Chi Squre Distribution

- therefore we conclude to say that Chi squre distibution has one important parameter that is degree of freedom and the mean and varaince is degree of freedom dependent

- *Home Work*: Derive an expression for Chi squre distribtion of degree 1

- *NOTE* : You need to gain more understanding on this topic by going beyond this elementary introduction

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