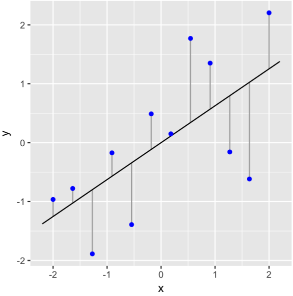
Curve Fitting to the Data



Ghulam Nabi Dar

Department of Physics KU Srinagar

August 7, 2020

Outline

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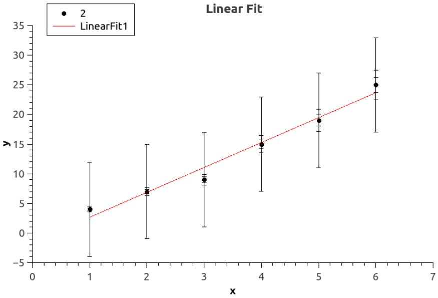

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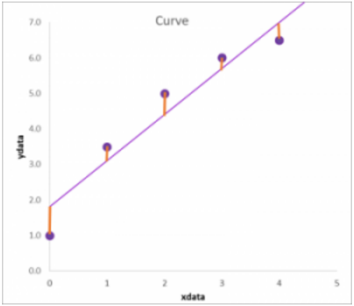

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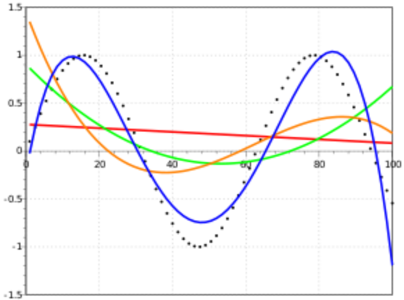

Polynomial curves fitting points generated with a sine function. The black dotted line is the "true" data, the red line is a first degree polynomial, the green line is second degree, the orange line is third degree and the blue line is fourth degree.

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- In the next unit we will go through another technique called *Interpolation*

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- More about this technique in the next chapter on *Numerical Analysis*.

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- such that the function $f(x)$ fits the data well

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- Mathematically we add up all the deviations and the line is considered the best fit having minimum error.

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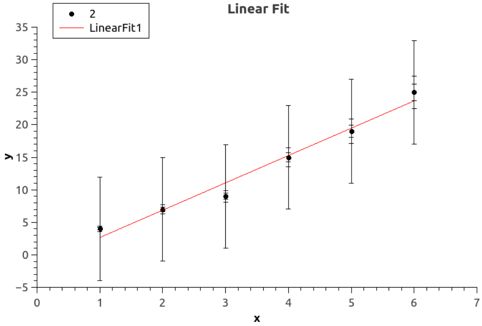

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- we can write them in matrix form as

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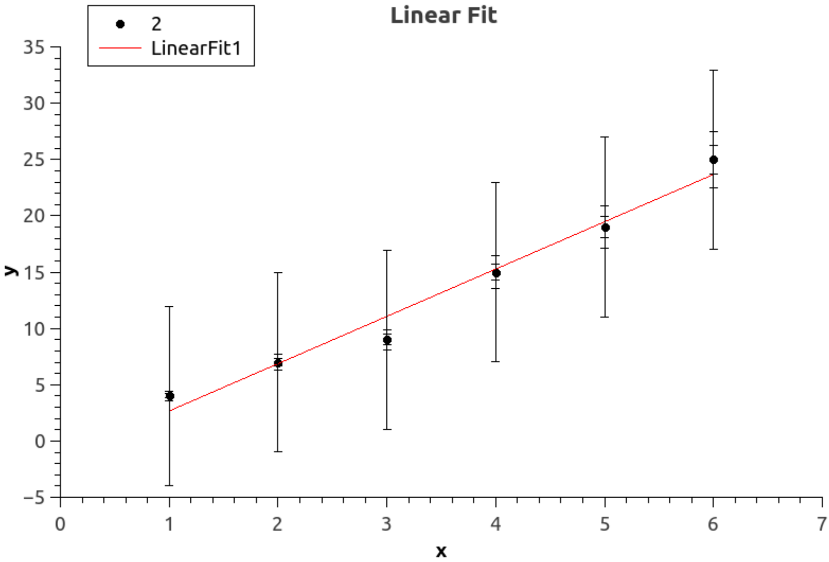

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- we can write it like a matrix equation as above and solve for coefficient matrix

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using given data and coefficients can be evaluted and hence the fitted line is the best fit line .
similarly one can proceed for third, fourth degree curve fitting

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- let the polynomial of j^{th} order fitted to given data set is

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- recall for linear we have two equations
- for parabolic three and

Polynomial Curve Fitting

- with respect a_j
- $$-2\sum_{i=1}^{n}\left[y_i-\left(a_0+\sum_{k=1}^{j}a_k x_i^k\right)\right]x_i^j=0$$
- there are $j+1$ equations
- recall for linear we have two equations
- for parabolic three and
- now for $j+1$ for j^{th} order
  we can write in matrix form and solve for coefficients as

Polynomial Curve Fitting

Polynomial Curve Fitting

- $$\begin{pmatrix} n & \sum x_i & \sum x_i^2 & ..... & \sum x_i^J \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & ..... & \sum x_i^{J+1} \\ . & . & . & ..... & . \\ . & . & . & ..... & . \\ . & . & . & ..... & . \\ \sum x_i^j & \sum x_i^{j+1} & \sum x_i^{j+2} & ..... & \sum x_i^{j+j} \end{pmatrix} * \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ a_j \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ . \\ . \\ \sum x_i^j y_i \end{pmatrix}$$

Polynomial Curve Fitting

- $$\begin{pmatrix} n & \sum x_i & \sum x_i^2 & ..... & \sum x_i^J \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & ..... & \sum x_i^{J+1} \\ . & . & . & ..... & . \\ . & . & . & ..... & . \\ . & . & . & ..... & . \\ \sum x_i^j & \sum x_i^{j+1} & \sum x_i^{j+2} & ..... & \sum x_i^{j+j} \end{pmatrix} * \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ a_j \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ . \\ . \\ \sum x_i^j y_i \end{pmatrix}$$

- $$A * X = B$$

Polynomial Curve Fitting

- $$\begin{pmatrix} n & \sum x_i & \sum x_i^2 & ..... & \sum x_i^J \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & ..... & \sum x_i^{J+1} \\ . & . & . & ..... & . \\ . & . & . & ..... & . \\ . & . & . & ..... & . \\ \sum x_i^j & \sum x_i^{j+1} & \sum x_i^{j+2} & ..... & \sum x_i^{j+j} \end{pmatrix} * \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ a_j \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ . \\ . \\ \sum x_i^j y_i \end{pmatrix}$$

- $$A * X = B$$

- we can write it like a matrix equation as above and solve for coefficient matrix

Polynomial Curve Fitting

- $$\begin{pmatrix} n & \sum x_i & \sum x_i^2 & ..... & \sum x_i^J \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & ..... & \sum x_i^{J+1} \\ . & . & . & ..... & . \\ . & . & . & ..... & . \\ . & . & . & ..... & . \\ \sum x_i^j & \sum x_i^{j+1} & \sum x_i^{j+2} & ..... & \sum x_i^{j+j} \end{pmatrix} * \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ a_j \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ . \\ . \\ \sum x_i^j y_i \end{pmatrix}$$
- $$A * X = B$$
- we can write it like a matrix equation as above and solve for coefficient matrix
- $$AX = B \rightarrow X = A^{-1} * B$$

Polynomial Curve Fitting

Polynomial Curve Fitting

- using given data

Polynomial Curve Fitting

- using given data
- coefficients can be evaluted

Polynomial Curve Fitting

- using given data
- coefficients can be evaluted
- and hence the fitted line is the best fit line

Some Problems on curve fitting

Some Problems on curve fitting

Example

a) For the given table , fit a steright line.

i	1	2	3	4	5	6
x	0	0.5	1.0	1.5	2.0	2.5
y	0	1.5	3.0	4.5	6.0	7.5

- *Solution*:
  We see from the given table

$$\sum x_i = 7.5, \sum x_i^2 = 13.75, \sum y_i = 22.5, \sum x_i y_i = 41.25$$

we know for linear fit..

Some Problems on curve fitting

Some Problems on curve fitting

- $$\begin{pmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{pmatrix} * \begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \end{pmatrix}$$

$$\begin{pmatrix} 6 & 7.5 \\ 7.5 & 13.75 \end{pmatrix} * \begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} 22.5 \\ 41.25 \end{pmatrix}$$

$$A * X = B$$

Some Problems on curve fitting

Some Problems on curve fitting

- we can write it like a matrix equation as above and solve for coefficient matrix

$$AX = B \rightarrow X = A^{-1} * B$$

we see

$$\begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

Some Problems on curve fitting

Some Problems on curve fitting

Example

b) For the given table , fit a sterigh line.

i	1	2	3	4	5	6
x	0	0.5	1.0	1.5	2.0	2.5
y	2	-0.43261	-0.1656	3.1253	4.7877	8.6909

-

Home Work

Home Work

Example

c) Fit a second order polynomial to the give data set

i	1	2	3	4	5	6
x	0	0.5	1.0	1.5	2.0	2.5
y	0	0.25	1.0	2.25	4.0	6.25

- *Solution*:
  For second order $j = 2$ that is $f(x) = a_0 + a_1 x + a_2 x^2$

Home Work

Home Work

- $$\begin{pmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{pmatrix} * \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{pmatrix}$$
  $$A * X = B$$

Home Work

Home Work

- we can write it like a matrix equation as above and solve for coefficient matrix

$$AX = B \rightarrow X = A^{-1} * B$$

using given data and coefficients can be evaluted and hence the fitted line is the best fit line .

Home Work

Home Work

- $$\begin{pmatrix} 6 & 7.5 & 13.75 \\ 7.5 & 13.75 & 8.125 \\ 13.75 & 28.125 & 61.1875 \end{pmatrix} * \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 13.75 \\ 28.125 \\ 61.1875 \end{pmatrix}$$

  $$A * X = B$$

  we can write it like a matrix equation as above and solve for coefficient matrix

  $$AX = B \rightarrow X = A^{-1} * B$$

- Using given data set

- Using given data set
- Coefficients can be evaluted using equations

- Using given data set
- Coefficients can be evaluted using equations
- Hence the fitted line is the best fit line .

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$