# Distributions and Parameters

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# Outline

- Distributions and parameters
- Examples of discrete,continous distribution

1. Parameters of Distribution
   - Expectation value E(X)
   - Mode
   - Median
   - Variance
   - Moments
2. Home work

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- Distributions are best known by parameters like expectation value, mode, median,variance,kutosis etc.

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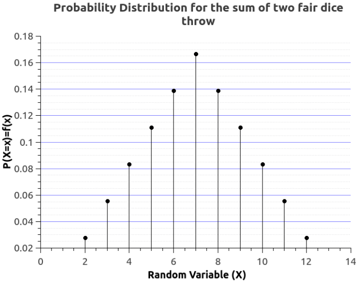


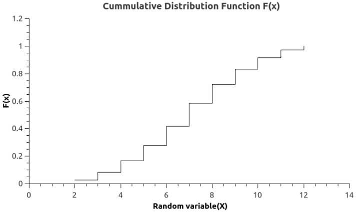

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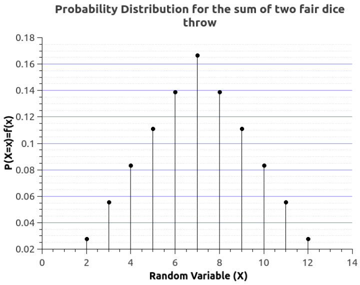


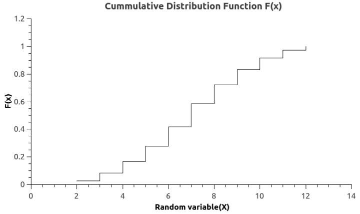


P(sum=7) is

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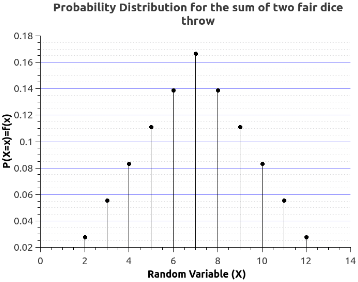


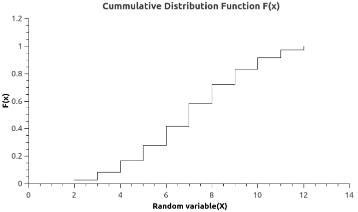


Discrete Distributions

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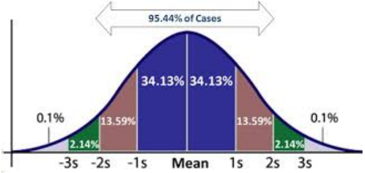


Continous Distribution

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*Note*: In the normal distribution $mean = mode = median = \mu$

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$$= \int_{-\infty}^{+\infty} x^3 f(x) dx - \mu_1 \int_{-\infty}^{+\infty} f(x) dx - 3\mu_1 \int_{-\infty}^{+\infty} x^2 f(x) dx + 3\mu_1^2 \int_{-\infty}^{+\infty} x f(x) dx$$

$$= \mu_3^3 - 3\mu_1\mu_2 + 2\mu_3$$

1. *Home Work*: Prove all the above results.
2. *Home Work*: Find constant $c$ and compute $P(1 < x2)$ when
3. f(x)= $\begin{cases} cx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$
4. Solution: Hint: First find $c$ using $f(x) \geq 0$ and $\int_{-\infty}^{+\infty} f(x)dx = 1$
5. *Home work*: Find $F(x)$ in above work. Find as given below

   F(x)= $\begin{cases} 0, & x < 0 \\ \frac{x^3}{27}, & 0 < 3 \\ 1, & x \geq 3 \end{cases}$