Contents

Probability Distributions

G.N.Dar

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0.1 Distributions and parameters

In *random experiments* we have seen random variables X both discrete and continous in nature. Both the variables take finite and infinite number of valuse x and corresponfing to each value is associated a Probability or Probability function $f(x) = P(X = x)$. This $f(x)$ verses x defines a distribution. There are number of distributions both discrete as well as continous corresponfing to the discrete rando variable as well as continous random variable. These distributions are best known by some parameters such as expectation value, mode, median,variance,kutosis etc.

0.2 Parameters of distribution

Expaction Value E(X):

Let

$$X = x_1, x_2, \cdots, x_n$$

And corresponfing Probabilities

$$p_1, p_2, p_3, \cdots, p_n$$

expectation value for discrete random variable case is s

$$E(X) = \mu = \sum_{i=1}^{n} x_i f(x_i)$$

where $f(x_i) = P(x_i)$, like $f(x_1) = P(X = x_1), f(x_2) = P(X = x_2)$ and soon

And for continous random variable case

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

Mode: It is value of random variable $X = x$ at which Probabilities function $f(x) = P(X = x)$ is maximum. In two dice throw mode if X represent the sum of the faces is 7

Median: It is the value of RV at which Cummulative Probability function $F(x)$ takes the value half $\frac{1}{2}$

In two dice throw expectation value if X represent the sum of the faces is 6,mode is 7 and median is x as

$$X = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$$

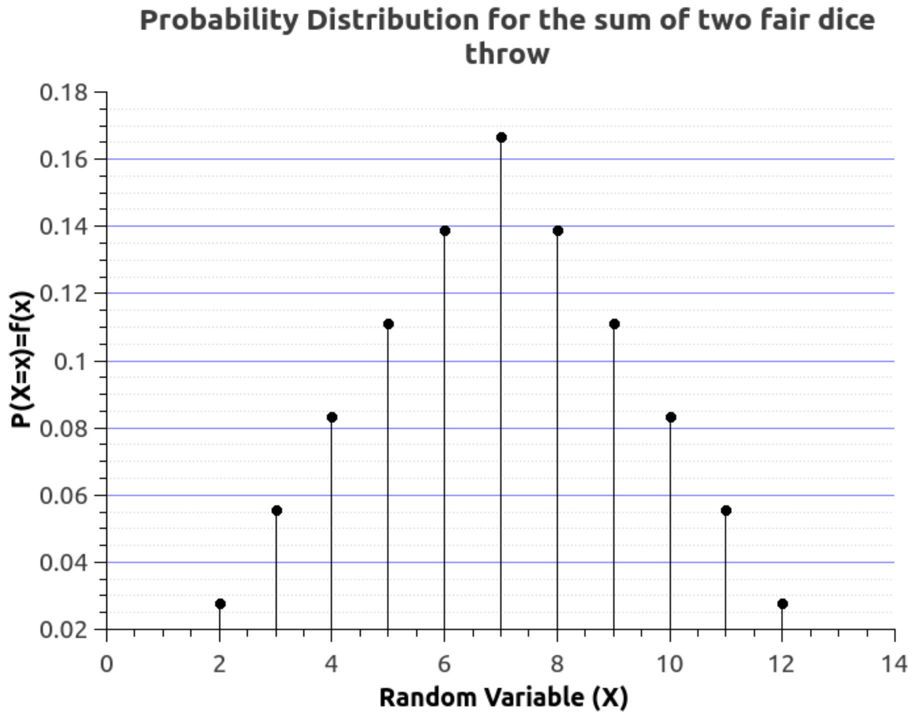

Figure 1: Discrete Distributions

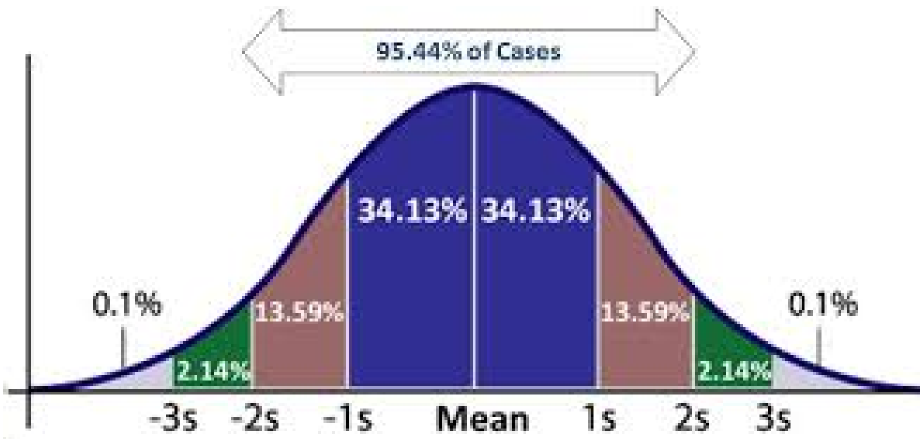

Figure 2: Continous Distribution

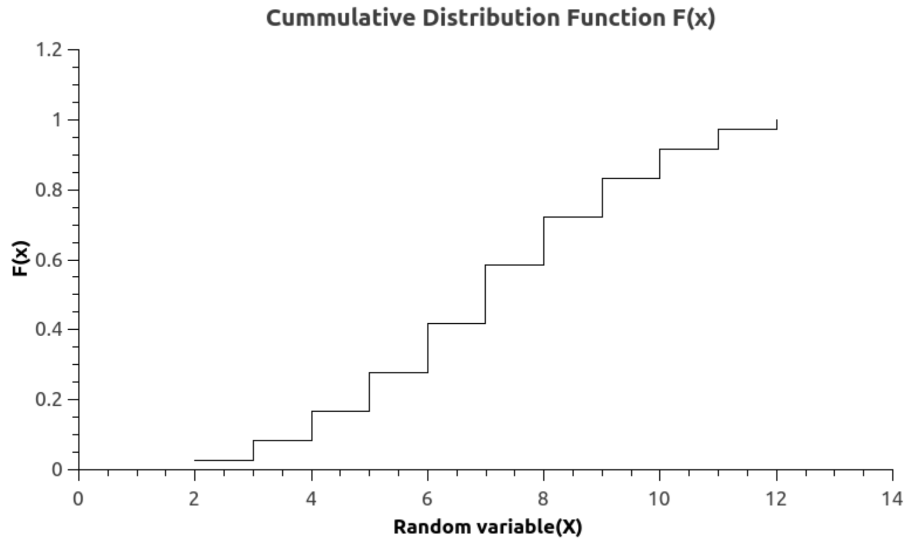

Figure 3: Discrete Distributions

Sum is presented here in tabular form also.

$\rightarrow$	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$f(x) = \frac{1}{36}, \frac{2}{36}, \frac{3}{36}, \frac{4}{36}, \frac{5}{36}, \frac{6}{36} \frac{5}{36} \frac{4}{36} \frac{3}{36} \frac{2}{36} \frac{1}{36}$$

OR

$$f(x) = 0.0278, 0.0556, 0.0834, 0.111, 0.139, 0.166666668, 0.1390.111, 0.0834, 0.0556, 0.0278$$

Same is ploted as shown in the figure.

$$E(X) = \mu = x.P(X = x) = x.f(x) = \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{22}{36} + \frac{12}{36} = \frac{222}{36} = 6$$

$$F(x) = \frac{1}{36}, \frac{3}{36}, \frac{6}{36}, \frac{10}{36}, \frac{15}{36}, \frac{21}{36}, \frac{26}{36}, \frac{30}{36}, \frac{33}{36}, \frac{35}{36}, \frac{36}{36} = 1$$

Mode is just 7

Note: In the normal distribution $mean = mode = median = \mu$

Variance(σ^2):

$$\sigma^2 = var(X) = E\left(x - E(X)\right)^2$$

Which can be solved

$$\sigma^2 = E\left[x^2 + \left(E(x)^2 - 2xE(X)\right)\right]$$

$$= E(X^2) + (E(x))^2 - 2E(X).E(X)$$

$$\sigma^2 = E(X^2) - E(X)^2 = E(X^2) - \mu^2$$

For discret and continous distribution the variance is defined as

$$\sigma^2 = E[(X-\mu)^2] = \sum_{i=1}^{n}(x_i - \mu)^2 f(x_i)$$

and

$$\sigma^2 = E[(X-\mu)^2] = \int_{-\infty}^{+\infty}(x-\mu)^2 f(x)dx$$

Similar to above parameters we can defined moments as first moment for discret

$$E(X) = \sum_{i=1}^{n} x_i f(x_i) = \mu_1$$

and for continous

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \mu_1$$

Similarly second moment

$$E(X^2) = \sum_{i=1}^{n} x_i^2 f(x_i) = \mu_2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \mu_2$$

$. \; . \; . \; . \; . \; . \; . \; . \; k^{th} moment$

$$E(X^k) = \sum_{i=1}^{n} x_i^k f(x_i) = \mu_k$$

$$E(X^k) = \int_{-\infty}^{\infty} x^k f(x)dx = \mu_k$$

Similarly varainces are defined as

$$\sigma^k = E[(X-\mu)^k] = \sum_{i=1}^{n}(x_i - \mu)^k f(x_i)$$

and

$$\sigma^k = E[(X-\mu)^k] = \int_{-\infty}^{+\infty} (x-\mu)^k f(x)dx$$

for $k = 1$ *we have*

$$\nu_1 = \sigma^1 = E[(X-\mu)^1] = \sum_{i=1}^{n} (x_i - \mu)^1 f(x_i) = 0$$

and

$$\nu_1 = \sigma^1 = E[(X-\mu)^1] = \int_{-\infty}^{+\infty} (x-\mu)^1 f(x)dx = 0$$

for $k = 2$ *defines the standard deviation given as*

$$\nu_2 = \sigma^2 = E[(X-\mu)^2] = \sum_{i=1}^{n} (x_i - \mu)^2 f(x_i)$$

and

$$\nu_2 = \sigma^2 = E[(X-\mu)^2] = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx$$

for $k = 3$ *defines the standard deviation given as*

$$\nu_3 = \sigma^3 = E[(X-\mu)^3] = \sum_{i=1}^{n} (x_i - \mu)^3 f(x_i)$$

$$= E[X^3 - \mu_1^3 - 3X^2\mu_1 + 3\mu_1^2 X] = \mu_3^3 - 3\mu_1\mu_2 + 2\mu_3$$

and

$$\nu_3 = \sigma^3 = E[(X-\mu)^3] = \int_{-\infty}^{+\infty} (x-\mu)^3 f(x)dx$$

$$= \int_{-\infty}^{+\infty} x^3 f(x)dx - \mu_1 \int_{-\infty}^{+\infty} f(x)dx - 3\mu_1 \int_{-\infty}^{+\infty} x^2 f(x)dx + 3\mu_1^2 \int_{-\infty}^{+\infty} x f(x)dx$$

$$= \mu_3^3 - 3\mu_1\mu_2 + 2\mu_3$$

Home Work*: Prove all the above results.*

Home Work*: Find constant* c *and compute* $P(1 < x2)$ *when*

$$f(x) = \left\{cx^2, \qquad 0 < x < 3\right\}$$
$$f(x) = \{0\}$$

otherwise Solution: Hint: First find c *using* $f(x) \geq 0$ *and* $\int_{-\infty}^{+\infty} f(x)dx = 1$
Home work*: Find* $F(x)$ *in above work. Find as given below*

$$F(x) = 0, x < 0$$

$$F(x) = \frac{x^3}{27}, 0 < 3$$

and

$$F(x) = 1, x \geq 3$$