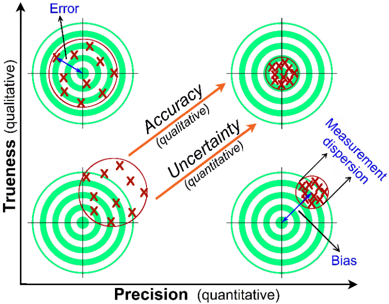
Error Propagation



Ghulam Nabi Dar
Department of Physics KU Srinagar

July 29, 2020

Outline

1. Error Propagation
   - **Some problems on Error propagation**
   - True Value

Error Propagation

Error Propagation

- Let us have measurements x and y

Error Propagation

- Let us have measurements x and y
- if we know the uncertainity in the varaible say x and y.

Error Propagation

- Let us have measurements x and y
- if we know the uncertainity in the varaible say x and y.
- if $Q(x, y)$ is the function of x and y

Error Propagation

- Let us have measurements x and y
- if we know the uncertainity in the varaible say x and y.
- if $Q(x, y)$ is the function of x and y
- The aim of this lecture is to know the uncertainity in function Q

Error Propagation

- Let us have measurements x and y
- if we know the uncertainity in the varaible say x and y.
- if $Q(x, y)$ is the function of x and y
- The aim of this lecture is to know the uncertainity in function Q
- The technique used is known as *Propogation of Errors*.

Error Propagation

Error Propagation

- AIM:: Varaince σ_Q^2 as a function of Varainces say σ_x^2 and σ_y^2
- if X measures some quantity with varaince σ_x^2
- and if Y with a variance σ_y^2,

Error Propagation

- AIM:: Varaince σ_Q^2 as a function of Varainces say σ_x^2 and σ_y^2
- if X measures some quantity with varaince σ_x^2
- and if Y with a variance σ_y^2,
- then the total varaice is given as

Error Propagation

- AIM:: Varaince σ_Q^2 as a function of Varainces say σ_x^2 and σ_y^2
- if X measures some quantity with varaince σ_x^2
- and if Y with a variance σ_y^2,
- then the total varaice is given as
- $$\sigma_Q^2 = \sigma_x^2 \left(\frac{\partial Q}{\partial x} \right)^2 \Bigg|_{\mu_x} + \sigma_y^2 \left(\frac{\partial Q}{\partial y} \right)^2 \Bigg|_{\mu_y}$$

Error Propogation

Error Propogation

- We know for measuring a quantity x say height of one single student in the class repeatedly we will get due to uncontrolable errors as

Error Propogation

- We know for measuring a quantity x say height of one single student in the class repeatedly we will get due to uncontrolable errors as
- $$x = \{x_1, x_2, x_3, x_4, x_1, x_5, x_{n-1}, x_n\}$$

Error Propogation

- We know for measuring a quantity x say height of one single student in the class repeatedly we will get due to uncontrolable errors as
- $$x = \{x_1, x_2, x_3, x_4, x_1, x_5, x_{n-1}, x_n\}$$
- similarly for any other quantity say y

$$y = \{y_1, y_2, y_3, y_4, y_1, y_5, y_{n-1}, y_n\}$$

Error Propogation

- We know for measuring a quantity x say height of one single student in the class repeatedly we will get due to uncontrolable errors as
- $$x = \{x_1, x_2, x_3, x_4, x_1, x_5, x_{n-1}, x_n\}$$
- similarly for any other quantity say y

$$y = \{y_1, y_2, y_3, y_4, y_1, y_5, y_{n-1}, y_n\}$$

- then the best value is taken as average given as

$$\bar{x} = \mu_x = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Error Propogation

Error Propogation

- $$\bar{y} = \mu_y = \frac{1}{n}\sum_{i=1}^{n} y_i$$

  As the measurements varaied so is the variation or error propagated into the function say we have

  $$f(x_1, y_1) \neq f(x_2, y_2) \neq \cdots \neq f(x_n, y_n) \neq f(\mu_x, \mu_y)$$

Error Propogation

- $$\bar{y} = \mu_y = \frac{1}{n}\sum_{i=1}^{n} y_i$$

  As the measurements varaied so is the variation or error propagated into the function say we have

  $$f(x_1, y_1) \neq f(x_2, y_2) \neq \cdots \neq f(x_n, y_n) \neq f(\mu_x, \mu_y)$$

- Therfore we define a parameter as

  $$Q_i = f(x_i, y_i) = Q_1, Q_2, Q_3.......Q_n$$

  for $i = 1, 2, 3, ..., n$

Error Propogation

- $$\bar{y} = \mu_y = \frac{1}{n}\sum_{i=1}^{n} y_i$$

  As the measurements varaied so is the variation or error propagated into the function say we have

  $$f(x_1, y_1) \neq f(x_2, y_2) \neq \cdots \neq f(x_n, y_n) \neq f(\mu_x, \mu_y)$$

- Therfore we define a parameter as

  $$Q_i = f(x_i, y_i) = Q_1, Q_2, Q_3 ....Q_n$$

  for $i = 1, 2, 3, ..., n$
- And we also define measurement at mean

  $$Q = f(\mu_x, \mu_y)$$

Error Propogation

Error Propogation

- Using Taylor series we expand a function Q_i about the mean μ assuming the measured values are close to the mean and *neglect higher order ters*,therefore

Error Propogation

- Using Taylor series we expand a function Q_i about the mean μ assuming the measured values are close to the mean and *neglect higher order ters*,therefore
-
$$Q_i = f(x_i, y_i) = f(\mu_x, \mu_y) + (x_i - \mu_x)\left(\frac{\partial Q}{\partial x}\right)\bigg|_{\mu_x} + (y_i - \mu_y)\left(\frac{\partial Q}{\partial y}\right)$$

- From the definition of variance (considering varaible here as Q instead of x) we know

$$\sigma_Q^2 = \frac{1}{n}\sum_{i=1}^{n}(Q_i - Q)^2$$

Error Propogation

Error Propogation

- $$Q_i - Q = (x_i - \mu_x)\left(\frac{\partial Q}{\partial x}\right)\Bigg|_{\mu_x} + (y_i - \mu_y)\left(\frac{\partial Q}{\partial y}\right)\Bigg|_{\mu_y} + higher \quad order$$

Error Propogation

- $$Q_i - Q = (x_i - \mu_x)\left(\frac{\partial Q}{\partial x}\right)\bigg|_{\mu_x} + (y_i - \mu_y)\left(\frac{\partial Q}{\partial y}\right)\bigg|_{\mu_y} + higher \quad order$$
- in fact $i = 1, 2...n$

Error Propogation

- $$Q_i - Q = (x_i - \mu_x)\left(\frac{\partial Q}{\partial x}\right)\bigg|_{\mu_x} + (y_i - \mu_y)\left(\frac{\partial Q}{\partial y}\right)\bigg|_{\mu_y} + higher \quad order$$
- in fact $i = 1, 2...n$
- $$\sum_{i=1}^{n}(Q_i - Q) = (x_i - \mu_x)\left(\frac{\partial Q}{\partial x}\right)\bigg|_{\mu_x} + (y_i - \mu_y)\left(\frac{\partial Q}{\partial y}\right)\bigg|_{\mu_y} + neglect$$

Error Propogation

Error Propogation

- Squring both sides

$$\frac{1}{n}\sum_{i=1}^{n}(Q_i - Q)^2 = \frac{1}{n}\sum_{i=1}^{n}\left((x_i - \mu_x)\left(\frac{\partial Q}{\partial x}\right)\bigg|_{\mu_x}\right)^2 + \sum_{i=1}^{n}\frac{1}{n}\left((y_i - \mu_y)\left(\frac{\partial Q}{\partial y}\right)\bigg|_{\mu_y}\right)^2$$

Error Propogation

Error Propogation

- $$\sigma_Q^2 = \sigma_x^2 \left(\frac{\partial Q}{\partial x}\right)^2\Bigg|_{\mu_x} + \sigma_y^2 \left(\frac{\partial Q}{\partial y}\right)^2\Bigg|_{\mu_y} + \frac{2}{n}\sum_{i=1}^{n}(x_i - \mu_x)(y_{-}\mu_y)\left(\frac{\partial Q}{\partial x}\right)\Bigg|_{\mu_x}\left(\frac{\partial Q}{\partial y}\right)\Bigg|_{\mu_y}$$

Error Propogation

Error Propogation

- Let us define

$$\sigma_{xy} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)(y_i - \mu_y)$$

Error Propogation

- Let us define

$$\sigma_{xy} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)(y_i - \mu_y)$$

-

$$\sigma_Q^2 = \sigma_x^2 \left(\frac{\partial Q}{\partial x}\right)^2\Bigg|_{\mu_x} + \sigma_y^2 \left(\frac{\partial Q}{\partial y}\right)^2\Bigg|_{\mu_y} + 2\sigma_{xy} \left(\frac{\partial Q}{\partial x}\right)\Bigg|_{\mu_x} \left(\frac{\partial Q}{\partial y}\right)\Bigg|_{\mu_y}$$

Error Propogation

Error Propogation

- **Some Cases**
  Case (i)
  If x and y are uncorrelated that is $\sigma_{xy} = 0$ we have

Error Propogation

- **Some Cases**
  Case (i)
  If x and y are uncorrelated that is $\sigma_{xy} = 0$ we have
- $$\sigma_Q^2 = \sigma_x^2 \left(\frac{\partial Q}{\partial x}\right)^2\Bigg|_{\mu_x} + \sigma_y^2 \left(\frac{\partial Q}{\partial y}\right)^2\Bigg|_{\mu_y}$$

Error Propogation

Error Propogation

- OR

$$\sigma_Q^2 = \sigma_x^2 f'^2(\mu_x) + \sigma_y^2 f'^2(\mu_y)$$

Case (ii)
If x and y are correlated that is then we have

Error Propogation

- OR

$$\sigma_Q^2 = \sigma_x^2 f'^2(\mu_x) + \sigma_y^2 f'^2(\mu_y)$$

Case (ii)
If x and y are correlated that is then we have

-

$$\sigma_Q^2 = \sigma_x^2 \left(\frac{\partial Q}{\partial x}\right)^2\Bigg|_{\mu_x} + \sigma_y^2 \left(\frac{\partial Q}{\partial y}\right)^2\Bigg|_{\mu_y} + 2\sigma_{xy} \left(\frac{\partial Q}{\partial x}\right)\Bigg|_{\mu_x} \left(\frac{\partial Q}{\partial y}\right)\Bigg|_{\mu_y}$$

Error Propogation

Error Propogation

- Case (iii)

Error Propogation

- Case (iii)
- If there is only one varaible involved say x

$$Q_i = f(x_i)$$

Error Propogation

- Case (iii)
- If there is only one varaible involved say x

$$Q_i = f(x_i)$$

-

$$Q = f(\mu_x)$$

Error Propogation

- Case (iii)
- If there is only one varaible involved say x

$$Q_i = f(x_i)$$

- $$Q = f(\mu_x)$$

- $$\mu_x = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$$

Error Propogation

- Case (iii)
- If there is only one varaible involved say x

$$Q_i = f(x_i)$$

- $$Q = f(\mu_x)$$

- $$\mu_x = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$$

- hen varaince or error in the varaible is

Error Propogation

- Case (iii)
- If there is only one varaible involved say x

$$Q_i = f(x_i)$$

- $$Q = f(\mu_x)$$

- $$\mu_x = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$$

- hen varaince or error in the varaible is
- $$\sigma_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Error Propogation

Error Propogation

- spread in the function or error propagated in the function is

Error Propogation

- spread in the function or error propagated in the function is
- $$\sigma_Q^2 = \sigma_x^2 \left(\frac{\partial Q}{\partial x}\right)^2 \Bigg|_{\mu_x} = \sigma_x^2 f'^2(\mu_x)$$

Some problems on Error propagation

Some problems on Error propagation

- Power in an electric circuit is given as $P = I^2R$ and $I = 1.0 \pm 0.1$ Ampere, $R = 10 \pm 1 Ohm$. Calculate the varaince in power using Propogation of error method.

Some problems on Error propagation

- Power in an electric circuit is given as $P = I^2R$ and $I = 1.0 \pm 0.1$ Ampere, $R = 10 \pm 1 Ohm$. Calculate the varaince in power using Propogation of error method.
- $$\sigma_P^2 = \sigma_I^2 \left(\frac{\partial P}{\partial I}\right)^2\bigg|_{\mu_I} + \sigma_R^2 \left(\frac{\partial P}{\partial R}\right)^2\bigg|_{\mu_R}$$

Some problems on Error propagation

Some problems on Error propagation

- $$\sigma_P^2 = \sigma_I^2 \left(\frac{\partial P}{\partial I} \right)^2 \Bigg|_{1.0} + \sigma_R^2 \left(\frac{\partial P}{\partial R} \right)^2 \Bigg|_{10}$$

Some problems on Error propagation

- $$\sigma_P^2 = \sigma_I^2 \left(\frac{\partial P}{\partial I}\right)^2\Bigg|_{1.0} + \sigma_R^2 \left(\frac{\partial P}{\partial R}\right)^2\Bigg|_{10}$$

- $$\sigma_P^2 = (0.1)^2(2IR) + (1)^2(I^2)^2$$

$$(0.1)^2(2.1.10)^2 + 1^2(1^2)^2 = 5watt$$

Some problems on Error propagation

Some problems on Error propagation

- Therfore

$$P = 10 \pm \sigma_P = 10 \pm 2$$

True power $P = 10watt$

Some problems on Error propagation

- Therfore

$$P = 10 \pm \sigma_P = 10 \pm 2$$

True power $P = 10 watt$

- But on repeated measurements we get with uncertainity of σ_P as per Gaussian distribution
68% measurement within $(8, 12)$ watts due to $(P \pm 1\sigma_P)$
95% measurement within $(6, 14)$ watts due to $(P \pm 2\sigma_P)$

Some problems on Error propagation

- Therfore
$$P = 10 \pm \sigma_P = 10 \pm 2$$
True power $P = 10 watt$
- But on repeated measurements we get with uncertainity of σ_P as per Gaussian distribution
68% measurement within $(8, 12)$ watts due to $(P \pm 1\sigma_P)$
95% measurement within $(6, 14)$ watts due to $(P \pm 2\sigma_P)$
- 99.7% measurement within $(4, 16)$ watts due to $(P \pm 3\sigma_P)$

Some problems on Error propagation

Some problems on Error propagation

- *Relative Error*

$$\frac{\sigma_P^2}{P^2} = \frac{\sigma_I^2}{P^2} 4.I^2 R^2 + \frac{\sigma_R^2}{P^2} I^2 = (0.1)^2$$

Some problems on Error propagation

- *Relative Error*

$$\frac{\sigma_P^2}{P^2} = \frac{\sigma_I^2}{P^2} 4.I^2 R^2 + \frac{\sigma_R^2}{P^2} I^2 = (0.1)^2$$

- $$= 4\frac{\sigma_I^2}{I^2} + \frac{\sigma_R^2}{R^2 I^2} = 4(\frac{0.1}{1})^2 + (\frac{1}{10 * 1})^2 = (0.1)^2$$

Some problems on Error propagation

- *Relative Error*

$$\frac{\sigma_P^2}{P^2} = \frac{\sigma_I^2}{P^2} 4.I^2 R^2 + \frac{\sigma_R^2}{P^2} I^2 = (0.1)^2$$

- $$= 4\frac{\sigma_I^2}{I^2} + \frac{\sigma_R^2}{R^2 I^2} = 4(\frac{0.1}{1})^2 + (\frac{1}{10 * 1})^2 = (0.1)^2$$

- uncertainity in power is dominated by uncertainity in current as it involved as squre so current has to be measured very carefully in order to reduce errors.

Some problems on Error propagation

Some problems on Error propagation

- *Example*
  Error in measuring area of a table
  Let

Some problems on Error propagation

- *Example*
  Error in measuring area of a table
  Let
-

$$x = 95.0 \pm 0.5cm$$

$$y = 190.0 \pm 0.5cm$$

$$\sigma_x = 0.5cm$$

$$\sigma_y = 0.5cm$$

Some problems on Error propagation

Some problems on Error propagation

- so the error in area is

$$\sigma_A^2 = \sigma_x^2 \left(\frac{\partial A}{\partial x}\right)^2 \Bigg|_{\bar{x}} + \sigma_y^2 \left(\frac{\partial A}{\partial y}\right)^2 \Bigg|_{\bar{y}}$$

Area is given as

$$A = x * y$$

$$(\frac{\partial A}{\partial x})|_{\bar{x}} = y = 190$$

Some problems on Error propagation

- so the error in area is

$$\sigma_A^2 = \sigma_x^2 \left(\frac{\partial A}{\partial x}\right)^2\Bigg|_{\bar{x}} + \sigma_y^2 \left(\frac{\partial A}{\partial y}\right)^2\Bigg|_{\bar{y}}$$

  Area is given as

$$A = x * y$$

$$(\frac{\partial A}{\partial x})|_{\bar{x}} = y = 190$$

- $$(\frac{\partial A}{\partial y})|_{\bar{y}} = x = 95$$

  therefore error in area is

Some problems on Error propagation

Some problems on Error propagation

- $$\sigma_A^2 = \sigma_x^2 \left(\frac{\partial A}{\partial x}\right)^2 \bigg|_{\bar{x}} + \sigma_y^2 \left(\frac{\partial A}{\partial y}\right)^2 \bigg|_{\bar{y}}$$

  $$= (0.5)^2[190^2 + 95^2] = 0.011cm$$

Some problems on Error propagation

- $$\sigma_A^2 = \sigma_x^2 \left(\frac{\partial A}{\partial x}\right)^2\Bigg|_{\bar{x}} + \sigma_y^2 \left(\frac{\partial A}{\partial y}\right)^2\Bigg|_{\bar{y}}$$

  $$= (0.5)^2[190^2 + 95^2] = 0.011cm$$

- *Example*
  Uncertainity in simple pendulum

  $$T = 2\pi\sqrt{(\frac{L}{g})}$$

  $$g = \frac{4.\pi^2 L}{T^2}$$

  Therfore the error in g is given as a function of error in L and T as

Some problems on Error propagation

Some problems on Error propagation

- $$\sigma_g^2 = \sigma_L^2 \left(\frac{\partial g}{\partial L}\right)^2 \Bigg|_{\bar{L}} + \sigma_T^2 \left(\frac{\partial g}{\partial T}\right)^2 \Bigg|_{\bar{T}}$$

Some problems on Error propagation

- $$\sigma_g^2 = \sigma_L^2 \left(\frac{\partial g}{\partial L}\right)^2\Bigg|_{\bar{L}} + \sigma_T^2 \left(\frac{\partial g}{\partial T}\right)^2\Bigg|_{\bar{T}}$$

- substitute the value of a partial differentiaiton we have the Relative error as

$$\frac{\sigma_g}{g} = \sqrt{(\frac{\sigma_L^2}{L^2} + 4\frac{\sigma_T^2}{T^2}}$$

a) If

$$a = b + c$$

show that

$$\sigma_a^2 = \sigma_b^2 + \sigma_c^2$$

Hint:

$$\sigma_a^2 = \sigma_b^2 \left(\frac{\partial a}{\partial b}\right)^2 + \sigma_c^2 \left(\frac{\partial a}{\partial c}\right)^2$$

Some problems on Error propagation

Some problems on Error propagation

- c) Show the error in m in the expression

$$m = -2.5 ln_{10}(\frac{F}{F_0})$$

is

$$\sigma_m^2 = (1.087)^2 (\frac{\sigma_F}{F})^2$$

Some problems on Error propagation

- c) Show the error in m in the expression

$$m = -2.5 ln_{10}(\frac{F}{F_0})$$

is

$$\sigma_m^2 = (1.087)^2(\frac{\sigma_F}{F})^2$$

- Hint:

$$\sigma_m^2 = \sigma_F^2 \left(\frac{\partial m}{\partial F}\right)^2_{\bar{F}}$$

Some problems on Error propagation

- c) Show the error in m in the expression

$$m = -2.5 ln_{10}(\frac{F}{F_0})$$

is

$$\sigma_m^2 = (1.087)^2 (\frac{\sigma_F}{F})^2$$

- Hint:

$$\sigma_m^2 = \sigma_F^2 \left(\frac{\partial m}{\partial F} \right)_{\bar{F}}^2$$

- where

$$\left(\frac{\partial m}{\partial F} \right)_{\bar{F}} = -\frac{2.5}{F}$$

Some problems on Error propagation

Some problems on Error propagation

- d) If $F = x + k$ show

$$\sigma_F = \sigma_x$$

e) If $F = x * y$ show

$$(\frac{\sigma_F}{F})^2 = (\frac{\sigma_x}{x})^2 + (\frac{\sigma_y}{y})^2$$

f) If $F = kx$ show

$$(\frac{\sigma_F}{F}) = (\frac{\sigma_x}{x})$$

Some problems on Error propagation

- d) If $F = x + k$ show

$$\sigma_F = \sigma_x$$

e) If $F = x * y$ show

$$(\frac{\sigma_F}{F})^2 = (\frac{\sigma_x}{x})^2 + (\frac{\sigma_y}{y})^2$$

f) If $F = kx$ show

$$(\frac{\sigma_F}{F}) = (\frac{\sigma_x}{x})$$

- g) If $F = x^n$ show

$$(\frac{\sigma_F}{F})^2 = n^2(\frac{\sigma_x}{x})^2$$

True Value

True Value

- Although the best value is considered and taken as mean of the observations. However even the mean value devietes from the true or exact value. That is mean value is not the final one and there is scope to improve even that.

  Let us have measurements of a quantity X as

$$X = X_1, X_2, X_3, X_4, ...X_n$$

AND

True Value

- Although the best value is considered and taken as mean of the observations. However even the mean value devietes from the true or exact value. That is mean value is not the final one and there is scope to improve even that.

  Let us have measurements of a quantity X as

  $$X = X_1, X_2, X_3, X_4, ...X_n$$

  AND

- $$\bar{X} = \mu = \frac{1}{n}(X_1 + X_2 + X_3 + \cdots + X_n)$$

True Value

True Value

- Deviation of each X_i from $\bar{X}$ is given as

True Value

- Deviation of each X_i from $\bar{X}$ is given as
- $$\sigma_{X_i^2} = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

  for example

  $$\sigma_{X_1^2} = \frac{1}{n}(X_1 - \bar{X})^2$$

  $$\sigma_{X_2^2} = \frac{1}{n}(X_2 - \bar{X})^2$$

  etc

True Value

- Deviation of each X_i from $\bar{X}$ is given as
- $$\sigma_{X_i}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

  for example

  $$\sigma_{X_1}^2 = \frac{1}{n}(X_1 - \bar{X})^2$$

  $$\sigma_{X_2}^2 = \frac{1}{n}(X_2 - \bar{X})^2$$

  etc
- Now let us find the Deviation associated to mean value . we have seen mean is a function of $X_1, X_2, ...X - n$ that is

$$\mu = \frac{1}{n}(X_1 + X_2 + X_3 + \cdots + X_n)$$

True Value

True Value

- therfore

True Value

- therfore
- $$\sigma_\mu^2 = \sigma{x_1^2}\left(\frac{\partial\mu}{\partial X_1}\right)^2 + \sigma{x_2^2}\left(\frac{\partial\mu}{\partial X_2}\right)^2 + \cdots + \sigma{x_1^n}\left(\frac{\partial\mu}{\partial X_n}\right)^n$$

  Assume
  $$\sigma{x_1^2} = \sigma{x_2^2} = \sigma{x_3^2}\ldots\ldots = \sigma{x_n^2} = \sigma^2$$

True Value

- therfore
- $$\sigma_{\mu}^{2} = \sigma{x_1^2}\left(\frac{\partial\mu}{\partial X_1}\right)^2 + \sigma{x_2^2}\left(\frac{\partial\mu}{\partial X_2}\right)^2 + \cdots + \sigma{x_1^n}\left(\frac{\partial\mu}{\partial X_n}\right)^n$$

  Assume

  $$\sigma{x_1^2} = \sigma{x_2^2} = \sigma{x_3^2}\ldots\ldots = \sigma{x_n^2} = \sigma^2$$

- therfore

$$\sigma_{\mu}^{2} = \sigma^2\left[\left(\frac{\partial\mu}{\partial X_1}\right)^2 + \left(\frac{\partial\mu}{\partial X_2}\right)^2 + \cdots + \left(\frac{\partial\mu}{\partial X_n}\right)^n\right]$$

True Value

True Value

- substitute the required value we get

$$\sigma_\mu^2 = \sigma^2 \left[\frac{1}{n^2} + \frac{1}{n^2} + \cdots + \frac{1}{n^2} \right] = n \frac{\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

$$\sigma_\mu = \frac{\sigma}{\sqrt{n}}$$

True Value

- substitute the required value we get

$$\sigma_\mu^2 = \sigma^2 \left[\frac{1}{n^2} + \frac{1}{n^2} + \cdots + \frac{1}{n^2} \right] = n\frac{\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

$$\sigma_\mu = \frac{\sigma}{\sqrt{n}}$$

- Hence from the above expression it is clear that there is Deviation or spread associated to the mean with an amount equal to $\sigma_\mu = \frac{\sigma}{\sqrt{n}}$, which can be improved upon if the number of observations n for the given quantity is increased. And clearly the spread or uncertainity $\sigma_\mu \to 0$ if $n \to 0$.