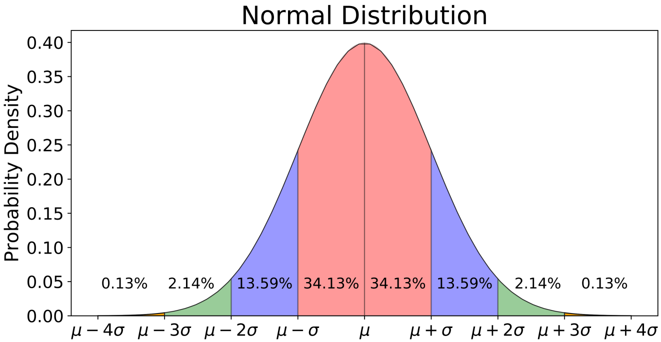
Normal Distribution and its Properties



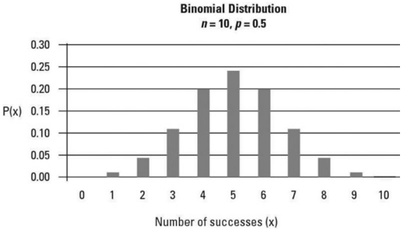
Ghulam Nabi Dar
Department of Physics KU Srinagar

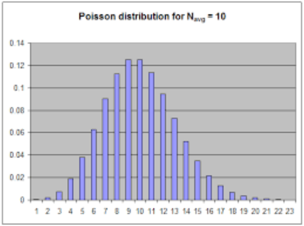
July 10, 2020

Outline

1. Normal Distribution (Continous Distribution)
2. Derivation of Normal Distribution from Poisson Distribution)
   - Expectation Value E(X)
   - Mode of Normal Distribution
   - Median of Normal Distribution M
   - Standard Normal Distribution
   - Numerical Problems on Normal Distribution

A comparision between the distributions





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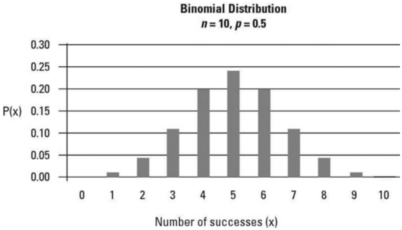

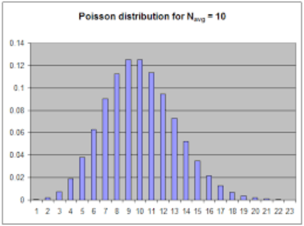

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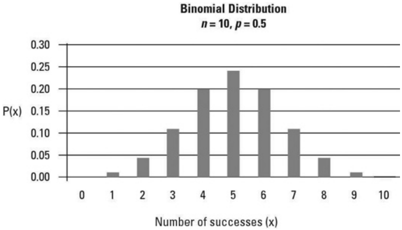

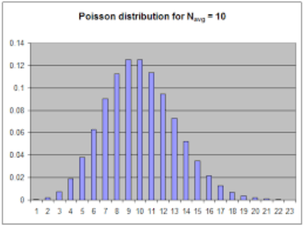

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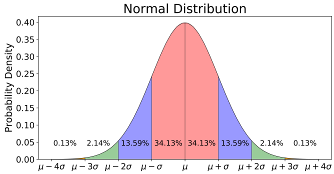
- 

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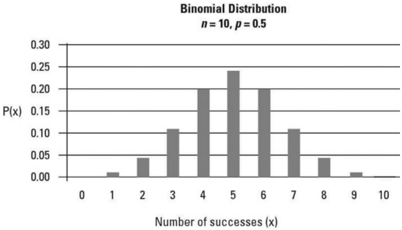

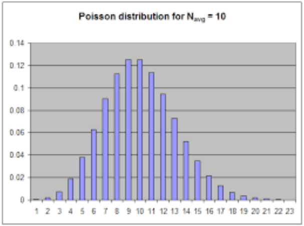

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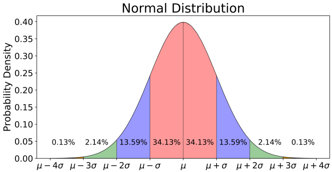
- 

- Normal:Continous Distribution

Normal Distribution

Probability Distribution $f(x)$ applied to a single variable but Continous in nature say heights of students in class, weights of new born babies, time taken to complete the task, typing speed of students in the class etc

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- $$f(x) = \frac{1}{\sigma.\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

  where σ is the variance and μ is the mean of the distribution.

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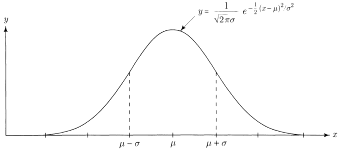
- 

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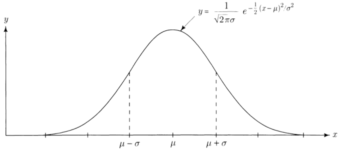
- 

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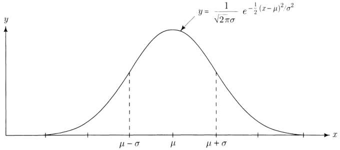
- 

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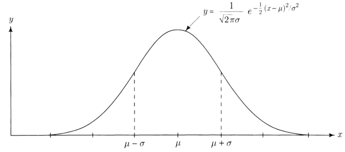
- 

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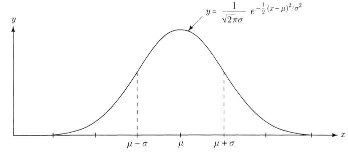
- 

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- approximately 68% data fall within $\pm 1\sigma$, 95% between $\pm 2\sigma$ and 99.7% between $\pm 3\sigma$

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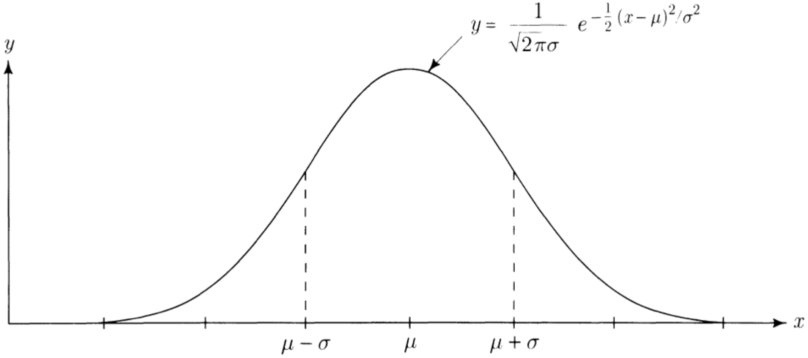
- 

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- which is the mean or expectation value of the normal distribution

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- from the formate it is easier to understand the function is odd one from $-\infty$ to ∞ and hence the result is surely zero.
- we can do in few steps as

$$I_1 = \int_{\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\ \sigma^2}} dx$$

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- Conclusion
  The mean mode and median of the normal distribution coincide and are equal to μ

Mean=Mode=Median $=\mu$

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- $$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = 1.\frac{\sqrt{\pi}}{2a^{\frac{3}{2}}}$$

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- $$\sigma^2 = \sigma^2$$

Numerical Problems on Normal Distribution

- Source: Library, Teaching and Learning-L.U. New Zealand
- 1. Potassium blood levels in healthy humans are normally distributed with a mean of 17.0 mg /100 ml, and standard deviation of 1.0 mg/100 ml. Elevated levels of potassium indicate an electrolyte balance problem, such as may be caused by Addison's disease. However, a test for potassium level should not cause too many "false positives".
  a) What level of potassium should we use so that only 2.5% of healthy individuals are classified as "abnormally high.

Numerical Problems on Normal Distribution

- 2. For a particular type of wool the number of ‘crimps per 10 cm’ follows a normal distribution with mean 15.1 and standard deviation 4.79.

Numerical Problems on Normal Distribution

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- (a) What proportion of wool would have a ‘crimp per 10 cm’ measurement of 6 or less?
  (b) If more than 7% of the wool has a ‘crimp per 10 cm’ measurement of 6 or less, then the wool is unsatisfactory for a particular processing. Is the wool satisfactory for this processing?

Numerical Problems on Normal Distribution

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- 3. The finish times for marathon runners during a race are normally distributed with a mean of195 minutes and a standard deviation of 25 minutes.

Numerical Problems on Normal Distribution

- 3. The finish times for marathon runners during a race are normally distributed with a mean of195 minutes and a standard deviation of 25 minutes.
- a) What is the probability that a runner will complete the marathon within 3 hours? b) Calculate to the nearest minute, the time by which the first 8% runners have completed the marathon. c) What proportion of the runners will complete the marathon between 3 hours and 4 hours?

Numerical Problems on Normal Distribution

4. The download time of a resource web page is normally distributed with a mean of 6.5 seconds and a standard deviation of 2.3 seconds.a) What proportion of page downloads take less than 5 seconds?b) What is the probability that the download time will be between 4 and 10 seconds? c) How many seconds will it take for 35% of the downloads to be completed?