Poisson Distribution

$$P(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Ghulam Nabi Dar

Department of Physics KU Srinagar

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Outline

1. Poisson Distribution
   - Poisson Distribution from Binomial Distribution
   - Expectation Value of Poisson Distribution E(X):
   - Varaince $\sigma^2(X)$
   - Poisson Distribution (Alternative Method)
   - Problems on Poisson Distribution (Home Work)

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- The Probability here is proportional to size of sample say region,time volume or number

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- $$f(x) = B(1, x) = \sum_{x=0}^{1} \frac{1!}{(1-x)!x!} p^x q^{1-x}$$
- with mean $np = 1.\frac{1}{2}$ and varaince $npq = 1.\frac{1}{2}\frac{1}{2} = \frac{1}{4}$

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- And for n it is $\frac{1}{2^n} \to 0$ for large n.

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- we can develop this Poisson Distribution independently as well as from Binomial as $n \to \infty$

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- $$f(x) = \frac{(np)^x}{x!}e^{-np}$$
- put $np = \lambda$ (mean of PD)

Poisson Distribution from Binomial Distribution

$$P(X = x) = f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Or we write a randam varaible follows a Poisson with mean λ as

$$P(x; \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

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- $$E(X) = e^{-\lambda}[\lambda + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \frac{\lambda^4}{3!} + ...]$$

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- so the expectation vaule of Poisson Distribution is λ

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- $$\sigma^2(x) = E(X-\mu)^2 = \sum_{x=0}^{n}(x-\mu)^2 P(x;\lambda) = \sum_{x=0}^{n}(x-\mu)^2 \frac{\lambda^x e^{-\lambda}}{x!}$$

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- $$E(X(X-1)) = \lambda^2 \sum_{y=0}^{m} \frac{e^{-\lambda} \lambda^y}{y!} = \lambda^2.1 = \lambda^2$$

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- Hence we see expectation as well as the varaince for Poisson Distribution is same as λ

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$t + \delta t$	zero event	$P(0;\delta t).P(0; = t)$
$t + \delta t$	x events	$P(x-1;t).P(1; = \delta t) + P(x;t).P(0; = \delta t)$

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- Total Probability in time δt

$$P(1, \delta t) + P(0, \delta t) = 1$$

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- $$\ln P(0, t) = -\lambda t + c$$

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- Multiply on both the sides by $e^{\lambda t}$

$$\frac{d}{dt}[e^{\lambda t}P(x;t)]=e^{\lambda t}P(x-1,t)$$

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- for $x=1$ the equation becomes as

- $$\frac{d}{dt}[e^{\lambda t}P(1;t)]=e^{\lambda t}P(0,t)$$

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- This is the desired Poisson Distribution with mean μ and varaince μ

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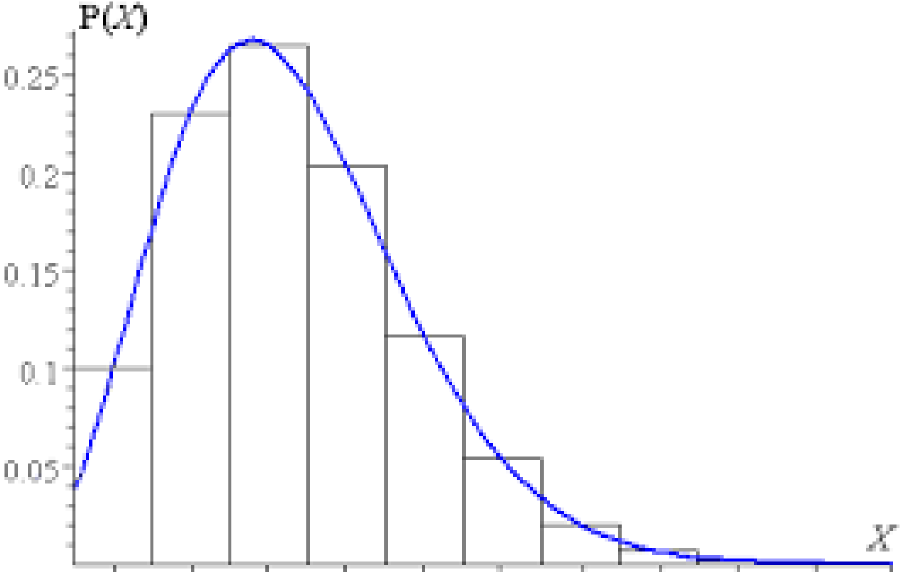
- we see the total Probability is one as

$$\sum_{x=0}^{\infty} P(x;\mu) = \sum_{x=0}^{\infty} \frac{\mu^x e^{-\mu}}{x!} = e^{-\mu} \sum_{x=0}^{\infty} \frac{\mu^x}{x!} = e^{-\mu} e^{\mu} = 1$$

Poisson Plots



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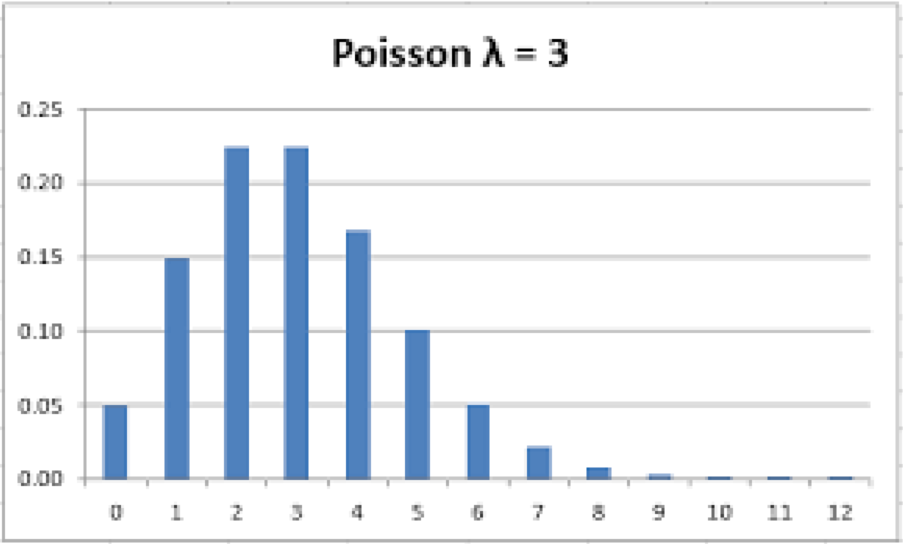
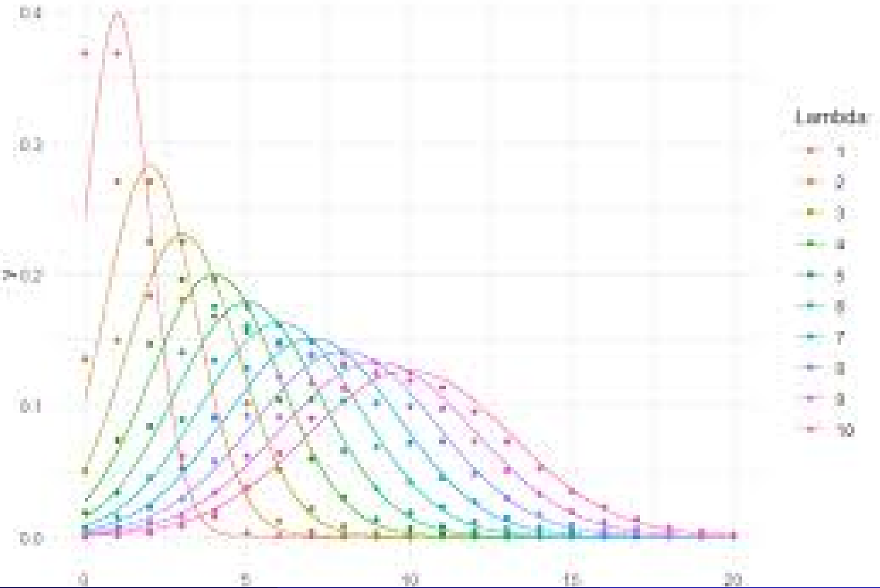


Poisson Plots



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- Now using complemnet

$$P(x \geq 4; 2) = 1 - P(x < 4; 2) = 1 - [P(x = 0; 2) + P(x = 1; 2) + P(x$$

$$= 1 - e^{-2}\left[\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!}\right] = 14.3\%$$

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- Solution: Here, $\lambda = 5$ and you need to Calculate $P(x = 3, 5)$. Answer: 14%

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- a) Calculate the probability that it emits 3 particles during a 5-second period.
- b) Calculate the probability that it emits at least one particle during a 5 second period.
- c) During a ten-second period, what is the probability that 6 particles are emitted?

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- 3. A 5-litre bucket of water is taken from a swamp. The water contains 75 mosquito larvae. A 200 mL flask of water is taken form the bucket for further analysis. What is
- a) the expected number of larvae in the flask? b) the probability that the flask contains at least one mosquito lava?

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- 4. If the light bulbs in a house fail according to a Poisson law, and over the last 15 weeks there have been 5 failures, find the probability that there will not be more than one failure next week.

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- 4. If the light bulbs in a house fail according to a Poisson law, and over the last 15 weeks there have been 5 failures, find the probability that there will not be more than one failure next week.
- 5. What is the probability of making 2 to 4 sales in a week if the average sales rate is 3 per week?